CHAPTER II

THEORETICAL APPROACH

Various relevant issues for this dissertation were reviewed for application in the analytical and empirical developments as studied herein. These study areas are categorized as prestressing concept and theory of plasticity.

2.1 Prestressing Concept

Typically, a prestressed concrete structure utilizes pre-loading in the tension regions to partially offset the actions of applied tensile stresses. But for active confinement purposes, we use prestressing forces in a somewhat different manner. The analytical concept employs the theory of plasticity which embodies the concepts of deviatoric stresses and yield surfaces as discussed in subsequent sections.

The present research is aimed at using conventional materials and techniques that are commonly used in construction works. Thus it was decided to use spiral prestressing strands for producing confining stress and pre-loading in the axial direction at the same time.

To create confining pressure by using prestressed strands, the curvature of the strand configuration produces uniform force along a strand and combining several strands around a concrete member will generate confined pressure as shown in Figure 2.1. The confining pressure can be calculated as shown below:

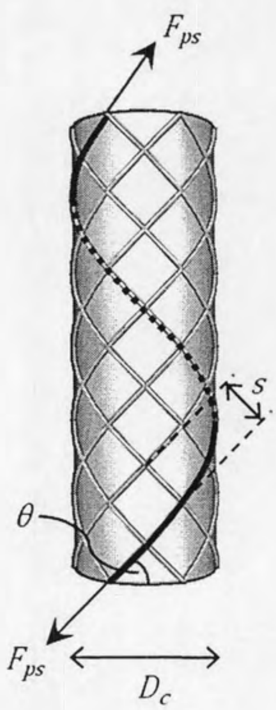

Figure 2.1 Strands alignment

$$f_l = \begin{cases} \dfrac{4A_{ps}f_{ps}\cos^2\theta}{D_c s} & \text{for cylinder section} \quad (2.1) \\ \dfrac{2A_{ps}f_{ps}\cos^2\theta}{ts} & \text{for pipe section} \quad (2.2) \end{cases}$$

$$F_a = nA_{ps}f_{ps}\sin\theta \quad (2.3)$$

$$f_a = \frac{F_a}{A_g} \quad (2.4)$$

$$\frac{f_a}{f_l} = 2\tan^2\theta \quad \text{for cylinder section} \quad (2.5)$$

$$s = \frac{2\pi D_c}{n}\sin\theta \quad (2.6)$$

where

f_l = the equivalent uniform confining pressure

f_{ps} = prestressing stress in strand

A_{ps} = cross area of prestress strand

D_c = diameter of core concrete measure from center of spiral strand

t = thickness of concrete pipe

s = spacing of prestress strand

n = number of strands

f_a = axial stress from applied prestress forces

An external post-tensioning system is suitable for applying spiral prestressing to a concrete member both from a theoretical as well as a practical point of view.

Tendon ducts can be easily fabricated in spiral alignment. Concrete members with outer post-tensioning tendons can develop fully confined areas with no spalling of the cover concrete. Such a system can be installed for retrofitting or strengthening purposes.

Tendon profiles may produce a large amount of prestress loss due to friction. To control such prestress losses, the radial dimensions of the concrete member and prestressing angle should be considered so as not to exceed the preferred amount. In this dissertation the expected amount of prestress losses due to friction does not exceed 30%. Friction losses can be calculated as follows:

$$\Delta\sigma_f = f_{ps}\left(1 - e^{-\mu_f \alpha_f - k_f L}\right) \tag{2.7}$$

where

$\Delta\sigma_f$	=	prestress loss due to friction
μ_f	=	curvature coefficient
α_f	=	curvature of strand
k_f	=	wobble coefficient
L	=	strand length

Creep due to prestressing forces must also be considered. Experimental evidence has indicated that the creep strain occurring over a given period is proportional to the applied stress. Research evidence, however, is conflicting with respect to the stress level at which the linearity between creep and applied stress ceases. Some research indicates a loss of linearity at compressive stresses as low as $0.2f_c'$ while other data suggest a value as high as $0.5f_c'$. However, the present research controls prestress forces not to exceed $0.2f_c'$ in order to reduce the nonlinearity of the creep problem.

For most systems of post-tensioning, when a tendon is tensioned, the jack is released and the prestress is transferred to the anchorage. The anchorage fixtures that are subject to stresses at this transfer point will tend to deform, thus allowing the tendon to slacken slightly. Friction wedges employed to hold the wires will slip a little distance before the wires can be firmly gripped. For short members, as for those used in this dissertation, loss due to anchorage take-up resulted in a large amount of prestress loss which was significant. In any case, loss due to anchorage take-up acts oppositely to friction which itself is also significant. Furthermore, losses over the length of a concrete member will compensate at each end and will be almost uniform throughout a member as shown in Figure 2.2.

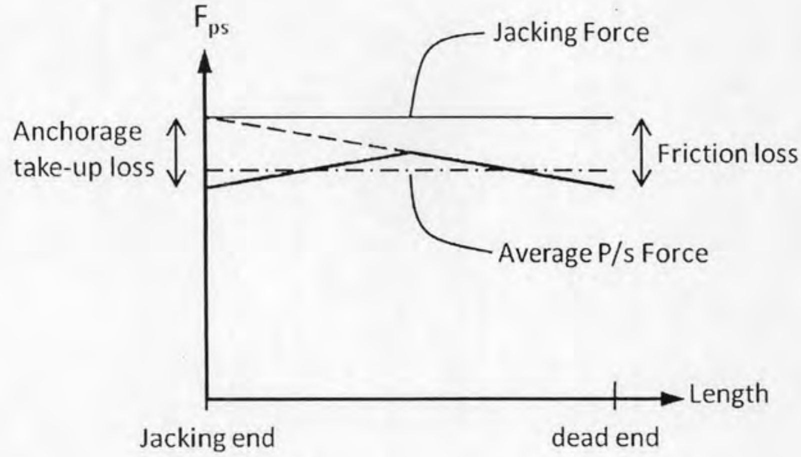

Figure 2.2 Prestress force along member length

2.2 Theory of Plasticity for Confinement

Confinement behavior can be determined by a theory of plasticity in terms of deviatoric stresses and yield criteria as illustrated in this section.

In general, strain increments may be regarded as being composed of recoverable (elastic) and non-recoverable (plastic) components,

$$\{d\boldsymbol{\varepsilon}\} = \{d\boldsymbol{\varepsilon}^{el}\} + \{d\boldsymbol{\varepsilon}^{pl}\} \tag{2.8}$$

where superscripts el and pl denote elastic and plastic, respectively. Stress increments are associated with only the elastic component.

$$\{d\sigma\} = [\mathrm{E}]\{d\boldsymbol{\varepsilon}^{el}\} \text{ or } \{d\sigma\} = [\mathrm{E}](\{d\boldsymbol{\varepsilon}\} - \{d\boldsymbol{\varepsilon}^{pl}\}) \tag{2.9}$$

where [E] is the elastic material property matrix. In a cylindrical coordinate system, $\{d\sigma\}$ contains increments of all six components of stress,

$$\{d\sigma\} = [d\sigma_{rr} \quad d\sigma_{\theta\theta} \quad d\sigma_{zz} \quad d\tau_{r\theta} \quad d\tau_{\theta z} \quad d\tau_{zr}]^T \tag{2.10}$$

The three essential components of elastic-plastic analysis are a yield criterion, a flow rule, and a hardening rule.

2.2.1 The Criterion of Yielding

The yield criterion relates the state of stress to the onset of yielding. Suppose that an element of material is subjected to a system of stresses of gradually increasing magnitude. The initial deformation of the element is entirely elastic and the original shape of the element is recovered on complete unloading. For certain critical combinations of the applied stresses, plastic deformation first appears in the element. A law defining the limit of elastic behavior under any possible combination of stresses is called a yield criterion. In developing a mathematical theory, it is necessary to take into account a number of idealizations at the outset. Firstly, it is assumed that the conditions of loading are such that a strain rate can be neglected. Secondly, the Bauschinger effect and the hysteresis loop, which arise from nonuniformity on the microscope scale, are disregarded. Finally, the material is assumed to be isotropic, so that its properties at each point are the same in all directions. There is a useful and immediate simplification resulting from the experimental fact that yielding is practically unaffected by a uniform hydrostatic tension or compression. The effects of these restrictions on the nature of the yield criterion may be represented in geometrical terms.

Consider a system of three mutually perpendicular axes with the principal stresses taken as Cartesian coordinates as illustrated in Figure 2.3.

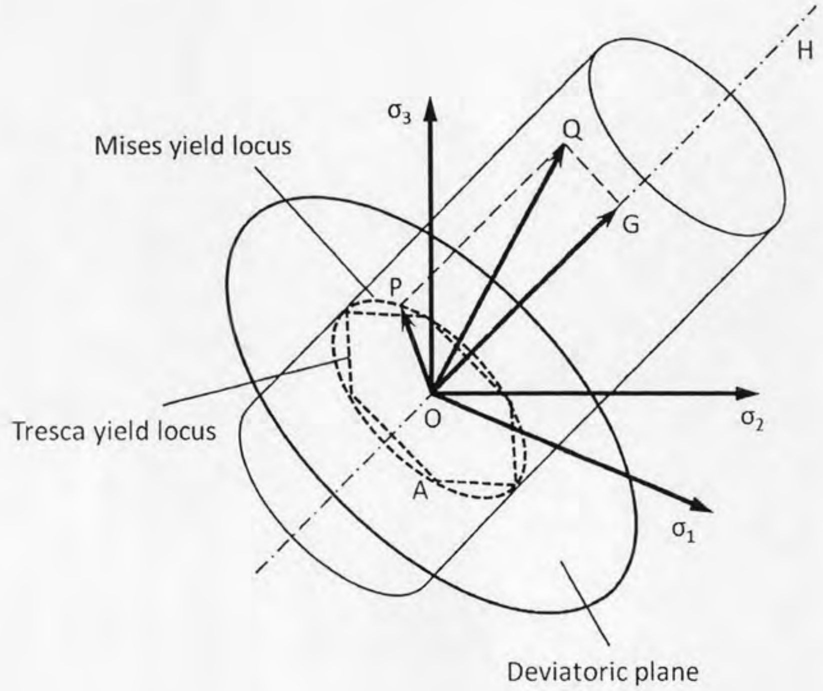

Figure 2.3 Geometrical representation of yield criterion in principal stress space

The state of stress at any point in a body may be represented by a vector emanating from the origin O. Imagine a line OH equally inclined to the three axes, so that its direction cosines are $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$. The stress vector **OQ**, whose components are $(\sigma_1, \sigma_2, \sigma_3)$, may be resolved into a vector **OG** along OH and a vector **OP** perpendicular to OH. The vector **OG** has a magnitude of $\sqrt{3}\sigma_o$ and represents the hydrostatic stress with components $(\sigma_o, \sigma_o, \sigma_o)$. The vector **OP** represents the deviatoric stress with components (s_1, s_2, s_3) and its magnitude is $\sqrt{2J_2}$ where J_2 is the second deviatoric stress invariant, $J_2 = \frac{1}{2} s_{ij} s_{ij}$. For any given state of stress, the deviatoric stress vector will lie in the plane passing through O and perpendicular to OH. This plane is known as the deviatoric plane and its equation is $\sigma_1 + \sigma_2 + \sigma_3 = 0$ in the principal stress space. Since a uniform hydrostatic stress has no effect on yielding, it follows that yielding can depend only on the magnitude and direction of the deviatoric stress vector OP. The yield surface is therefore a right cylinder whose generators are perpendicular to the deviatoric plane. Any point inside the cylinder represents an elastic state of stress. The curve A in which the yield surface is intersected by the deviatoric plane is called the yield locus. The equation to a possible yield locus, which is assumed to be convex, is a possible yield criterion.

Consider a concrete member confined with lateral stresses such as P_1 and P_2 and principal stress σ_3 in the axial direction throughout the concrete member as shown in Figure 2.4. If we increase confining stresses σ_1 and σ_2, the stress σ_3 in the axial direction will increase proportionally as illustrated in Figure 2.5.

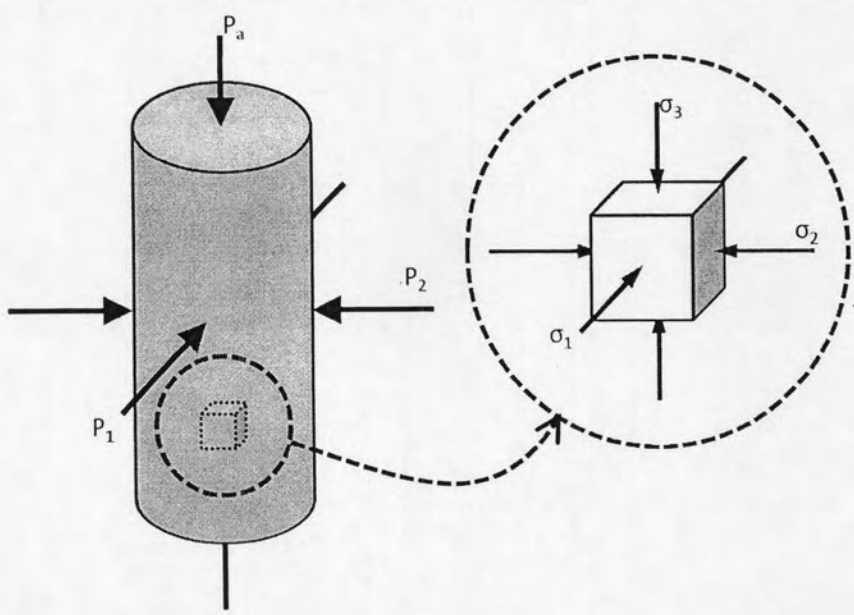

Figure 2.4 Principal stresses on the element of concrete member under confined stresses

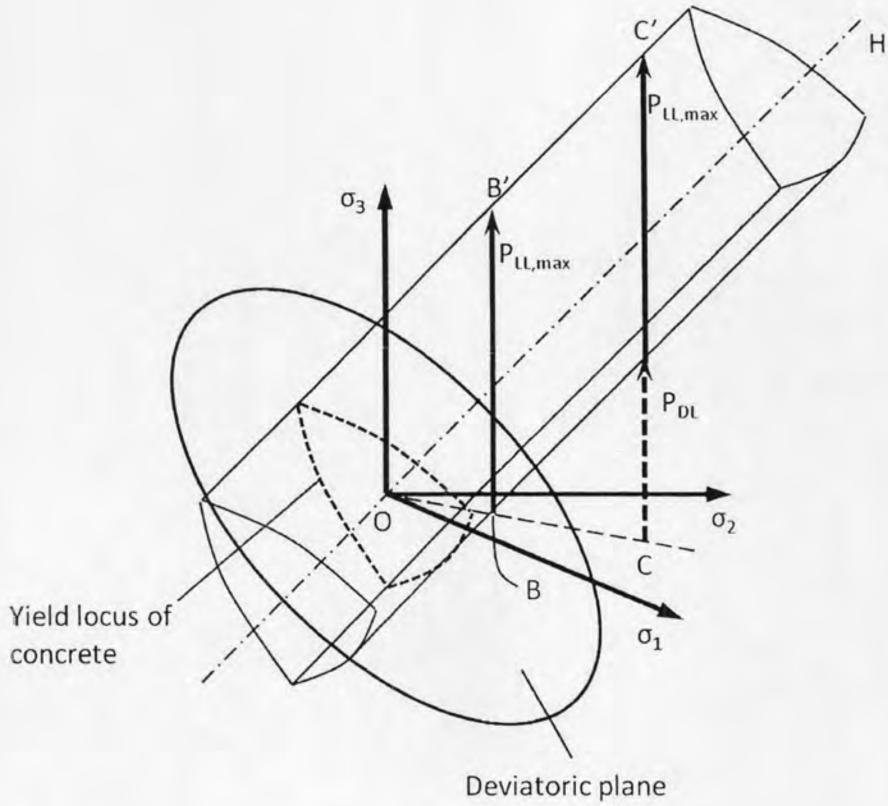

Figure 2.5 State of stresses corresponding to yield criterion of concrete

Figure 2.5 illustrates, through plastic theory, confinement effects on the strength of the concrete at each state of stress. At point B, there are confining stresses σ_1 and σ_2, and axial stress $\sigma_3 = 0$ on the yield surface . This is a maximum state for confining stresses without pre-axial load. $P_{LL,max}$ is a magnitude of axial strength of concrete under the confined stress state at point B. If we want to increase axial strength, σ_3 , confining stresses need to be shifted from point B along direction BC. Axial stress component,σ_3 results from pre-loading or self–weight so-called static dead load, P_{DL} combined with active load as $P_{LL,max} + P_{DL}$. For a realistic structure there exists a very heavy dead load. Regardless of the dead load of the structure, we can provide a proper amount of confining stresses to maintain a state of stress within the yield surface such that the material remains elastic. Theoretically, such a structure can resist a value of dead load which may increase without limit. Beyond the consideration of plastic theory, however, one must consider many factors which may affect the strength of concrete. These include buckling due to slender geometry and creep due to large amounts of confining stresses.

2.2.2 Strain-Hardening Postulates

An element of material yields when the magnitude of the deviatoric stress vector is increased to a value such that the stress point reaches the yield locus and the strain vector is directed outwards beyond the locus. The magnitude of the stress vector causing yielding depends on its final direction in the deviatoric plane. If the material is non-hardening, the plastic stress state can change in such a way that the stress point always lies on a constant yield locus. For a strain-hardening material, the size and shape of the yield locus depend on the complete history of plastic deformation.

Consider first the hypothesis in which the amount of hardening is taken as a function of the total plastic work per unit volume. This assumption is obviously consistent with the fact that no hardening is produced by purely elastic strains. If the plastic part of the strain increment tensor is denoted by $d\varepsilon_{ij}^{pl} = \dot{\varepsilon}_{ij}^{pl} dt$, where $\dot{\varepsilon}_{ij}^{pl}$ is a measure of the rate of deformation and dt is the time element, the increment of plastic work per unit volume is

$$dW_p = \sigma_{ij} d\varepsilon_{ij}^{pl} = \left(s_{ij} + \sigma_o \delta_{ij}\right) d\varepsilon_{ij}^{pl} = s_{ij} d\varepsilon_{ij}^{pl} \qquad (2.11)$$

where δ_{ij} is identity matrix

Now consider hardening rules that account for anisotropy and Bauschinger effect exhibited by experiment. It is assumed that the material is initially isotropic, having identical yield stresses in tension and compression. In the kinematic hardening rule, due to Prager[5], the yield surface is assumed to undergo translation in a nine-dimensional stress space. The initial yield surface is represented by the equation $f\left(\sigma_{ij}\right) = k^2$, where k is a constant. If the resultant displacement of the

[5] Prager, W. "A New Method of Analyzing Stress and Strains in Work-Hardening Plastic Solids." Journal of Applied Mechanics, 23, 1956.

yield surface at any stage is denoted by a symmetric tensor α_{ij}, the current yield surface is given by

$$f(\sigma_{ij} - \alpha_{ij}) = k^2 \tag{2.12}$$

since α_{ij} is not a scalar multiple of the isotropic tensor δ_{ij}, which represents a hydrostatic change in stress, the material becomes anisotropic as a result of the hardening process. It is reasonable to suppose that the incremental translation of the yield surface is in the direction of the plastic strain increment $d\varepsilon_{ij}^{pl}$, considered as a vector in the nine-dimensional space. Then

$$d\alpha_{ij} = c\, d\varepsilon_{ij}^{pl} \tag{2.13}$$

where c is a scalar quantity characterizing the material behavior. The deformation is assumed small, so that the effect of rotation of the element on $d\alpha_{ij}$ may be disregarded. Since $d\alpha_{ij} = 0$, the translation of the yield surface is always parallel to the deviatoric hyperplane.

The hardening rule describes how the yield criterion is modified by straining beyond initial yield. Let the yield function be written as:

$$F = F(\{\sigma\}, \{\alpha\}, W_p) \tag{2.14}$$

where $\{\alpha\}$ and W_p account for hardening by describing how a yield surface in multidimensional stress space is altered, by changes in location or size, in response to plastic strains as stated in equations (2.15) and (2.16). Elastic conditions prevail when $F < 0$ and when stresses are such that $F = 0$, yielding impends or is in progress. The case $F > 0$ is not physically possible. Starting from the state $F = 0$, plastic flow is associated with changes in $\{\sigma\}$ and changes in $\{\alpha\}$ and/or W_p. During plastic flow, stresses remain on the yield surface (which may be changing in shape and/or location as required by the hardening rule); hence $dF = 0$. When there is unloading, $dF < 0$, which signals a return to elastic behavior.

Hardening can be modeled as isotropic or as kinematic, either separately or in combination. Isotropic hardening can be represented by plastic work per unit volume W_p, which describes growth of the yield surface. Kinematic hardening can be represented by a vector $\{\alpha\}$, which accounts for translation of the yield surface in stress space. Those relations would be presented symbolically as:

For isotropic hardening: $$W_p = \int \{\sigma\}^T \{d\varepsilon^{pl}\} \tag{2.15}$$

For kinematic hardening: $$\{\alpha\} = \int [C] \{d\varepsilon^{pl}\} \tag{2.16}$$

where $[C] = \frac{2}{3} H_p \begin{bmatrix} 1 & 1 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

In general, the plastic modulus, H_p is not constant. It can be related to stress or strain by experiment. The diagonal matrix $[C]$ is not a unit matrix because we write $\{d\varepsilon\}$ using the engineering definition of shear strain rather than the tensor definition. In cylindrical coordinate, plastic flow takes place at constant volume,

$d\varepsilon_{rr}^{pl} + d\varepsilon_{\theta\theta}^{pl} + d\varepsilon_{zz}^{pl} = 0$; hence $[1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0]\{\alpha\} = \alpha_{rr} + \alpha_{\theta\theta} + \alpha_{zz} = 0.$

2.2.3 The Rule of Plastic Flow

When an element of material is unloaded from a certain plastic state, it recovers elasticity and the stress point moves inside the yield locus. If anisotropy is disregarded, the elastic behavior of the material is characterized by two independent elastic constants which retain their initial values. When the element is reloaded along a certain strain-path, yielding will again occur when the stress point reaches the current yield locus. For a work-hardening material (isotropic hardening material), a further plastic flow can be enforced only by increasing the stress to a point outside the yield locus. If the stress increment is such that the stress point remains on the same yield locus, no hardening is produced and the plastic strain increments are zero. Such changes in stress are called neutral since they represent neither loading nor unloading. The elastic part of the strain increment corresponding to any plastic flow is directly related to the stress increment by means of Hooke's law. It is therefore necessary to relate the increment of plastic strain to the stress increment and the current stress.

The flow rule relates the state of stress $\{\boldsymbol{\sigma}\}$ to the corresponding six increments of plastic strain $\{d\boldsymbol{\varepsilon}^{pl}\}$ when an increment of plastic flow occurs.

The flow rule is stated in terms of a function G, which has units of stress and is called a "plastic potential". With $d\lambda$ a scalar that may be called a "plastic multiplier", plastic strain increments are given by:

$$\{d\boldsymbol{\varepsilon}^{pl}\} = \left\{\frac{\partial G}{\{d\sigma\}}\right\} d\lambda \tag{2.17}$$

Thus $\{d\varepsilon_{rr}^{pl}\} = (\partial G/\partial \sigma_{rr})d\lambda$, and so on. The flow rule is called "associated" if $G = F$ and "nonassociated" otherwise. Non-associated rules are better suited for concrete material.

For concrete material, the model makes use of the yield function of Lubliner et. al.[6] (1989), with the modifications proposed by Lee and Fenves[7] (1998) to account for different evolution of strength under tension and compression. The evolution of the yield surface is controlled by the hardening variables, $\tilde{\varepsilon}_t^{pl}$ and $\tilde{\varepsilon}_c^{pl}$. In terms of effective stresses, the yield function takes the form:

$$F = \frac{1}{1-\alpha}\left(\overline{q} - 3\alpha\overline{p} + \beta(\tilde{\varepsilon}^{pl})\langle\widehat{\overline{\sigma}}_{\max}\rangle - \gamma\langle-\widehat{\overline{\sigma}}_{\max}\rangle\right) - \overline{\sigma}_c\left(\tilde{\varepsilon}_c^{pl}\right) = 0$$

with

$$\alpha = \frac{(\sigma_{bo}/\sigma_{co})-1}{2(\sigma_{bo}/\sigma_{co})-1}; 0 \leq \alpha \leq 0.5$$

$$\beta = \frac{\overline{\sigma}_c\left(\tilde{\varepsilon}_c^{pl}\right)}{\overline{\sigma}_t\left(\tilde{\varepsilon}_t^{pl}\right)}(1-\alpha) - (1+\alpha)$$

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[6] Lubliner, J., J. Oliver, S. Oller, and E. Oñate, "A Plastic-Damage Model for Concrete," International Journal of Solids and Structures, vol. 25, pp. 299–329, 1989.

[7] Lee, J., and G. L. Fenves, "Plastic-Damage Model for Cyclic Loading of Concrete Structures," Journal of Engineering Mechanics, vol. 124, no.8, pp. 892–900, 1998.

$$\gamma = \frac{3(1-K_c)}{2K_c-1}$$

Here,

$\hat{\bar{\sigma}}_{\max}$	is the maximum principal effective stress
σ_{bo}/σ_{co}	is the ratio of initial equibiaxial compressive yield stress to initial uniaxial compressive yield stress (the suitable value for concrete is 1.16)
K_c	is the ratio of the second stress invariant on the tensile meridian, $q_{(\mathrm{TM})}$, to that on the compressive meridian, $q_{(\mathrm{CM})}$, at initial yield for any given value of the pressure invariant p such that the maximum principal stress is negative, $\hat{\sigma}_{\max} < 0$ as shown in Figure 2.6. It must satisfy the condition $0.5 < K_c \leq 1.0$. The appropriate value for concrete as used in this thesis is $2/3$)
$\overline{\sigma}_t\left(\tilde{\varepsilon}_t^{pl}\right)$	is the effective tensile cohesion stress
$\overline{\sigma}_c\left(\tilde{\varepsilon}_c^{pl}\right)$	is the effective compressive cohesion stress

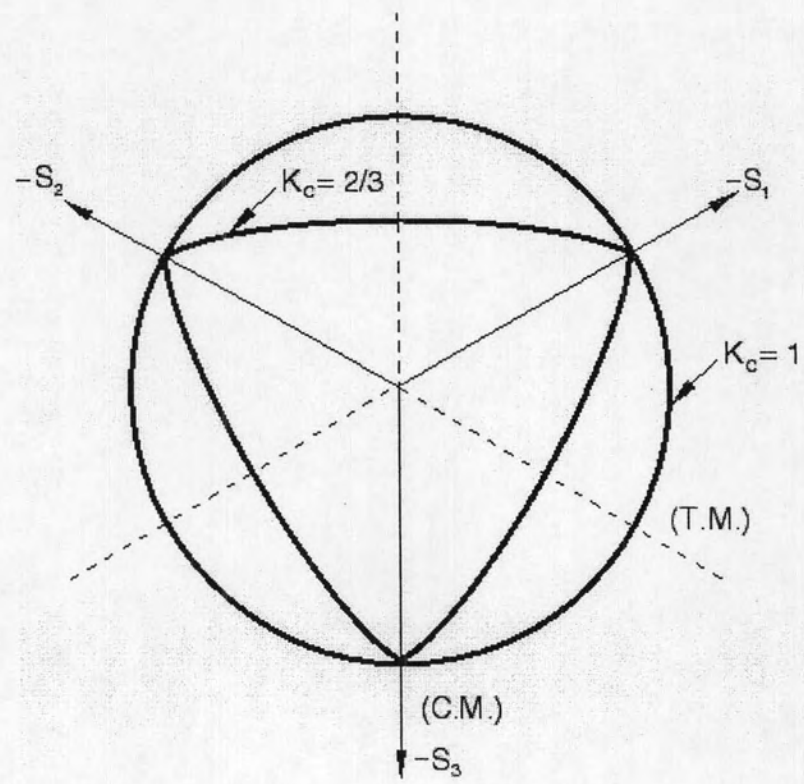

Figure 2.6 Yield surfaces in the deviatoric plane, with various values of K_c