ระเบียบวิธีไฟไนต์เอลิเมนต์รูปหลายเหลี่ยมเพื่อการวิเคราะห์คุณลักษณะแถบช่องความถี่ในผลึกโฟโตนิกส์สองมิติ

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A FINITE ELEMENT METHOD WITH POLYGONAL ELEMENTS FOR ANALYZING BAND GAP CHARACTERISTIC IN TWO-DIMENSIONAL PHOTONIC CRYSTALS

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วิทยานิพนธ์นำเสนอระเบียบวิธีไฟไนต์เอลิเมนต์เพื่อวิเคราะห์คุณลักษณะแถบช่องความถี่ในผลึกโฟโตนิกส์สองมิติโดยใช้ฟังก์ชันการประมาณแบบวาชเพรส (Wachspress) สำหรับเอลิเมนต์รูปหลายเหลี่ยม ได้มีการเขียนโปรแกรมไฟไนต์เอลิเมนต์ตามที่เสนอในวิทยานิพนธ์และใช้โปรแกรมที่เขียนมาคำนวณในกรณีตัวอย่างพร้อมกับเปรียบเทียบผลการคำนวณกับวิธีไฟไนต์เอลิเมนต์มาตรฐานที่ใช้เอลิเมนต์สามเหลี่ยมและวิธีกระจายคลื่นระนาบ (plane wave expansion) เพื่อตรวจสอบความถูกต้องของระเบียบวิธีที่นำเสนอ กรณีตัวอย่างที่นำเสนอในวิทยานิพนธ์คือผลึกโฟโตนิกส์ที่มีโครงผลึกรูปสามเหลี่ยมและสี่เหลี่ยมจัตุรัส ผลการเปรียบเทียบคำตอบระหว่างวิธีที่นำเสนอกับวิธีไฟไนต์เอลิเมนต์มาตรฐาน พบว่าวิธีที่นำเสนอให้ความแม่นยำดีกว่าบนพื้นฐานของตัวแปรที่เท่ากัน

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ENY SUKANI RAHAYU: A FINITE ELEMENT METHOD WITH POLYGONAL ELEMENTS FOR ANALYZING BAND GAP CHARACTERISTIC IN TWO-DIMENSIONAL PHOTONIC CRYSTALS, THESIS ADVISOR: ASST. PROF. TUPTIM ANGKAEW, D. Eng., 103 PP.

Finite element method (FEM) based on Wachspress interpolation on polygonal elements for analyzing the band gap characteristic of two-dimensional photonic crystals (2D PCs) is proposed in this thesis. The proposed FEM has been coded into program and validated with other method such as conventional finite element method based on triangular elements and plane wave expansion method. Numerical examples on triangular lattice and square lattice photonic crystals have been carried out. The comparison of accuracy between the conventional FEM and the proposed method has been also provided in this thesis. The comparison results show that the polygonal element can yield better accuracy than the conventional FEM with the same number of unknowns in system equations.

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