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Power Comparisons for the Test of Independence

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ABSTRACT

The empirical Type I error rates and power are investigated for five statistics for the test of independence: the Pearson χ^2 *with a chi-squared approximation, the likelihood ratio test statistic* G^2 *with a chi-squared approximation, Cressie and Read's statistic (C-R), Mielke and Berry's statistic (T), and Zelterman's statistic (V) in sparse r x c tables. Simulation results show that four statistics: the Pearson* χ^2*, C-R, T and V are valid with respect to Type I error rates. In most cases, the likelihood ratio test statistic* G^2 *is liberal.*

For power comparisons, in a small table (2 x 2), there is almost no differences in power among the four statistics: χ^2*, C-R, T, and V when sample sizes are large. Differences are found only when sample sizes are small and alpha is large. In these conditions, the power of the V statistic is lowest, and power of the remaining statistics are similar. In a larger table, 2 x 5, the T statistic is the most powerful, and power of the V, C-R and Pearson* χ^2 *statistics are similar.*

Introduction

Although the Pearson χ^2 test has been frequently used for almost a century in contingency table analyses for tests of independence, there is still no commonly accepted rule regarding the meaning of small expectation. Several applied statistics textbooks generally recommend the use of the χ^2 tests of independence with r x c tables only when expected frequency of each cell is five or more (Kirk, 1978, p. 341; Chao, 1974, p. 299). Recent research has shown this general rule is unneccessarily restrictive (Radlow & Alf, 1975; Camilli & Hopkins, 1978; Bradley et al., 1979). A controversy among statisticians has caused a problem for the applied researcher who must decide whether it is appropriate or not to use the Pearson χ^2.

Consequently, a number of new alternatives to the Pearson χ^2 have been developed. Several researchers (i.e., Hoeffding, 1965; Cohen & Sackrowitz, 1975) have compared χ^2 and G^2 in a variety of research situations. From these comparisons, neither of the two procedures emerges as a clear favorite. When one method is better in some respect than the other, it seems to result from a particular configuration of sample size, number of categories, and expected frequencies. Furthermore, Hosmane (1987) found that the Cressie and Read statistic (C-R) is as efficient or more efficient than the Pearson χ^2 when alpha = .01. In the same year, Zelterman introduced another new statistic, V, which is another adjusted Pearson χ^2 version. It has been hoped that this statistic would improve the validity and power of the test. In 1988, Berry and Mielke found that among the χ^2, G^2, and a nonasymptotic χ^2 (T) statistic is superior in overall performance according to Type I error rate control. The evidence provided so far indicate that alternatives may be more valid than the Pearson χ^2. Past investigations on alternatives to the Pearson χ^2 statistic for independence have focussed only on Type I error rates. Very little research has been conducted to examine power. Also, a review of literature has failed to identify a single study which examined both Type I error rates and power of the new alternative tests along with the Pearson χ^2 test.

Purpose

The purpose of the current study was to investigate the Type I error rates and the statistical power of the Pearson χ^2 statistic and four alternatives, the likelihood ratio test statistic G^2, the Cressie and Read statistic (C-R), the nonasymptotic χ^2 (T) suggested by Mielke and Berry, and the nonasymptotic χ^2 statistic (V) suggested by Zelterman. Based on the results of the study, recommendations on the following questions will be offered:

a) Under what conditions is the Pearson χ^2 test of independence appropriate?

b) Do the recent statistics provide valid tests in an r x c table?

c) Do the recently developed statistics improve power?

d) Can one procedure be recommended as a general solution to the problem of tests for independence when tables are sparse?

Method

For this Monte Carlo study, the data were created using SAS UNIFORM (SAS Version 6.08) to generate uniform random numbers on the interval from zero to one. For the test of independence, neither the row nor column marginal frequencies were fixed. Only total sample size (N) was fixed.

For the true null hypotheses, each condition, three significance levels (.01, .05, .10), four sample sizes (20, 50, 100, 300) for 2 x 2, 2 x 4, and 2 x 5 and two sample sizes (100, 300) for 4 x 5 contingency tables were investigated. Three marginal probability distributions ranging from uniform to highly skewed on both rows and columns for each 2 x 2, 2 x 4, and 2 x 5 table and five marginal probability distributions for each 4 x 5 table were examined. For each condition, 10,000 samples were repeated.

Insert Table 1 about here

In 10,000 replications in which the expected cell frequencies were very small, the random number generating process resulted in a row or column marginal with zero frequency; hence the χ^2 statistic was not computable. In this situation, that sample was re-generated and replaced.

A two standard error conficence interval, $alpha \pm 2\sqrt{\{alpha(1 - alpha) / number\ of\ replications\}}$, where alpha is the nominal Type I error rate, is used to evaluate the Type I error rate.

To simulate data for the r x c contingency table, a pseudorandom number generator sampled N numbers between 0 and 1 from a uniform distribution, and each number was used to place a count in one of the cells of the r x c table via reference to the joint cumulative probability distritution. Each cell in a given table was assigned by a cumulative proportion distribution from the cell proportions, for the r, c cells: [0, p1), [p1, p1 + p2), ..., [1 - p_{rc}, 1), respectively.

For example, in a 2 x 2 table with equal marginals and an effect size (ϕ) of zero (null hypothesis is true), the probability of obtaining an observation in cell (i j) equals the product of the corresponding marginal probabilities $p(RC_{ij}) = p(R_i)\ p(C_j)$.

Random numbers between 0 and .25 were assigned to the first row and first column, cell (1 1); random numbers between .26 and .50 were assigned to the first row and second column, cell (1 2); those between .51 and .75 were assigned to the second row and first column, cell (2 1); and those between .76 to 1.00 were assigned to the second row and second column, cell (2 2). This table would have expected proportion in each cell of. 25.

For power analysis, in the current study, statistical power is considered for three effect sizes, small (phi or C approximatedly equals .10), medium (phi or C approximatedly equals .25), and large (phi or C approximatedly equals .40) (Cohen, 1988, pp. 215-227).

In the non-null case, observations are assigned to table cells in proportions that different from the products of the table's marginal probabilities. In other words, the probability of obtaining an observation in cell (i j) equals the product of the corresponding probabilities incremented or decremented by Δp, that is

$$p(RC_{ij}) = p(R_i)\ p(C_j) + \Delta p,$$

where Δp is the amount to produce the degrees of association. For a 2 x 2 table according to Bradley & Cutcomb (1977),

$$\Delta p = \phi \sqrt{\frac{abcd}{bcd + acd + abd + abc}}$$

where a = $p(R_1)\ p(C_1)$, b = $p(R_1)\ p(C_2)$, c = $p(R_2)\ p(C_1)$, and d = $p(R_2)\ p(C_2)$. The probability increment / decrement factor $\Delta p = p(RC_{ij}) - p(R_i)\ p(C_j)$ is the size of the difference required in each cell to produce the amount of association ϕ entered in the formula. Note that Δp is the same for all four cells in 2 x 2 tables, with Δp being added to the diagonal cells and subtracted from the off diagonal cells in order to produce the desired distribution of joint probabilities and to ensure that they sum (within rows and columns) to the corresponding marginal probabilities.

In a 2 x 2 table with equal marginals with effect size (ϕ) of .4, Δp equals .1. Random numbers between 0 to .35 were assigned to the first row and first column, cell (1 1); those between .36 to .50 were assigned to cell (1 2); those between .51 to .65 were assigned to cell (2 1); and those between .66 to 1.00 were assigned to cell (2 2).

For an r x c table, according to Cohen (1988) the contingency coefficient (C) is defined

$$C = \sqrt{\frac{W^2}{W^2 + 1}}$$

$$W^2 = \sum_{i=1}^{m} \frac{(p_{1i} - p_{0i})^2}{p_{0i}} ,$$

where p0i = the proportion in cell i posited by the null hypothesis,

P1i = the proportion in cell i posited by the alternative hypothesis and reflected the effect for that cell,

m = the number of cells.

For a 2 x 5 contingency table, to obtain a given effect size, the joint probability p0i was adjusted by Δp in order to get the corresponding p_{1i}. Unlike 2 x 2 tables, for a 2 x 5 table, Δp has no specific formula. In this case Δp is determined by trial and error, then is added and subtracted at any row or column such that marginal row and column probabilities remain the same. For each condition, statistical power of the valid statistics was estimated across the 10,000 samples.

Result

Simulation results reveal that the four statistics: the Pearson χ^2 C-R, T, and V are valid with respect to Type I error rates. In general, the empirical Type I error rate of the C-R statistic tends to be closer to the nominal significance level than the Pearson χ^2. Both the Pearson χ^2 and the C-R statistic were conservative when all e_{ij} are less than five for the uniform marginal distribution and when more than 60% of e_{ij}'s are less than five for the moderately skewed marginal distribution. However for the extremely skewed marginal distribution, the Pearson χ^2 is inconsistent, that is, the Pearson χ^2 is not always conservative when more than 60% of e_{ij}'s are less than five whereas the C-R statistic is conservative in most cases.

Insert Table 2, 3, 4 and 5 about here

For all three distributions, the likelihood ratio test statistic G^2 is liberal and therefore is not valid with respect to Type I error rates. The T and V statistics have empirical Type I error rates in the acceptable range for all conditions examined.

For statistical power analysis in 2 x 2 tables, there are no differences among the four statistics: the Pearson χ^2, C-R, T, and V for the large sample sizes (100, 300). However for a small sample size (20) and large alpha (.10), the C-R statistic and the Pearson χ^2 are more powerful (2%) than the T and V statistics for the uniform marginal distribution. In these conditions, the T statistic and the Pearson χ^2 are more powerful (4%) than the C-R and V statistics for moderately and extremely skewed marginal distributions. For all three distributions, power of the V is lowest.

Insert Table 6,7 and 8 about here

For 2 x 5 tables, when N is small (20), the T and V statistics are more powerful (3%) than the C-R statistic and the Pearson χ^2 for uniform and moderately skewed marginal distributions. The T statistic and the Pearson (2 are more powerful (2%) than the C-R and V statistics for the extremely skewed marginal distribution. Power of the four statistics are similar for large sample sizes (100, 300).

Insert Table 9,10 and 11 about here

Discussion

1. Under what conditions is the Pearson χ^2 test of independence appropriate?

Based on the study's findings, the recommendations on the use of Pearson χ^2 are: (a) the Pearson χ^2 is robust in a sparse table (a table with less than 60% of the e_{ij} are less than five and none of them are less than one). The empirical Type I error rates of the Pearson χ^2 lie within the two standard error range, (b) the Pearson χ^2 tends to be conservative in a moderately sparse table (a table with more than 60% of the e_{ij} are less than five and none of them are less than one), and (c) the Pearson χ^2 tends to be too liberal and is not recommended in an extremely sparse table (a table with a large variation in the e_{ij} and more than 60% of the e_{ij} are less than five and some of them are less than one).

In contrast to Slakter's recommendations in 1966, the results of the current study indicate that when all e_{ij} are small but equal, the Pearson χ^2 is conservative. Furthermore, the results support Lewontin and Felsentein (1965); Camilli and Hopkins (1978); Bradley et al. (1979) conclusions that e_{ij} could be relaxed to one. However, if more than 60% of the e_{ij} are relaxed to one, the test would be conservative.

Therefore, the Pearson χ^2 is useable for a wider rang of the e_{ij} than had previously been suggested. In addition, the results from the current study support Camilli & Hopkins (1978) conclusion that in a 2 x 2 table if one or two cells have e_{ij} less than five, the Pearson χ^2 is still valid with respect to Type I error rates.

2. Do the recent statistics provide valid tests in an r x c table?

The likelihood ratio test statistic G^2 is liberal for all three marginal distributions and therefore is not valid for the conditions studied here.

The C-R statistic has empirical Type I error rates in the two standard error range for the sparse table, tends to be conservative for the moderately sparse table, and tends to be more conservative for the extremely sparse table.

The findings from this study do not support Cressie and Read (1984) and Bedrick (1987) results in that the C-R statistic was better than the Pearson χ^2 statistic in small sample size cases with respect to Type I error rates (see table 4). The finding also does not agree with Hosmane's findings reported in 1987 (see table 4). Hosmane concluded that the C-R statistic was as or more powerful than the Pearson χ^2 statistic when alpha = .01. The results from the current study indicate that no clear difference in power is found between the C-R and the Pearson χ^2 statistics.

The T and the V statistics have empirical Type I error rates within the two standard error range for all three marginal distributions. In general, the T statistic was superior among the four statistics in Type I error rate control for all conditions studied, and the T works well for sparse, moderately sparse, and extremely sparse tables.

3. Do the recently developed statistics improve statistical power?

No differences in power is found among the four statistics: the Pearson χ^2, C-R, T and V when sample size is large. However, in a small sample case and large table (2 x 5), the T statistic is more powerful than the Pearson χ^2. Power of the V, C-R, and Pearson χ^2 statistics are similar.

4. Can one procedure be recommended as a general solution to the problem of tests for independence when tables are sparse?

The T statistic is best in Type I error rate control for all three marginal distributions, and power of the T statistic outperforms all alternatives in five out of six, or 83% of conditions studied. Based on conditions investigated in the present study, the T statistic is recommended for testing independence between two qualitative variables.

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Table 1 Maginal row and column probabilities used in 2 x 2, 2 x 4, 2 x 5, 4 x 5 contingency tables

Table	Row				Column				
	1	2	3	4	1	2	3	4	5
2 × 2	.5	.5	-	-	.5	.5	-	-	-
	.7	.3	-	-	.8	.2	-	-	-
	.8	.2	-	-	.9	.1	-	-	-
2 × 4	.5	.5	-	-	.25	.25	.25	.25	-
	.7	.3	-	-	.4	.2	.2	.2	-
	.8	.2	-	-	.5	.2	.2	.1	-
2 × 5	.5	.5	-	-	.2	.2	.2	.2	.2
	.7	.3	-	-	.3	.2	.2	.2	.1
	.8	.2	-	-	.4	.2	.2	.1	.1
4 × 5	.25	.25	.25	.25	.2	.2	.2	.2	.2
	.4	.2	.2	.2	.3	.2	.2	.2	.1
	.5	.2	.2	.1	.4	.2	.2	.1	.1
	.6	.2	.1	.1	.6	.1	.1	.1	.1
	.8	.1	.05	.05	.6	.2	.1	.05	.05

Table 2 Empirical Type I error rates of the five statistics for uniform case Alpha

Alpha	Table	N	χ^2	G^2	C-R	T	V	e_{ij}
.01	2 × 2	20	.010	.012	.010	.010	.010	5
		50	.009	.011	.009	.009	.009	12.5
		100	.012	.013	.012	.012	.012	25
		300	.010	.010	.010	.010	.010	75
	2 × 4	20	.004	.021	.006	.010	.010	2.5
		50	.009	.015	.010	.010	.010	6.25
		100	.008	.010	.008	.009	.008	12.5
		300	.011	.011	.011	.011	.011	37.5
	2 × 5	20	.004	.021	.005	.009	.009	2
		50	.008	.018	.009	.010	.010	5
		100	.009	.011	.009	.010	.009	10
		300	.010	.011	.010	.010	.010	30
	4 × 5	100	.008	.017	.008	.009	.009	5
		300	.009	.011	.009	.010	.010	15
.05	2 × 2	20	.050	.056	.055	.050	.051	5
		50	.057	.059	.057	.052	.052	12.5
		100	.056	.056	.056	.056	.056	25
		300	.051	.051	.051	.050	.050	75
	2 × 4	20	.042	.090	.048	.053	.051	2.5
		50	.048	.062	.052	.049	.050	6.25
		100	.051	.055	.052	.051	.051	12.5
		300	.051	.053	.051	.051	.051	37.5
	2 × 5	20	.034	.102	.041	.049	.049	2
		50	.050	.073	.054	.053	.053	5
		100	.048	.057	.050	.049	.050	10
		300	.051	.053	.051	.051	.051	30
	4 × 5	100	.045	.073	.047	.046	.047	5
		300	.050	.056	.051	.051	.051	15
.10	2 × 2	20	.121	.121	.122	.110	.116	5
		50	.108	.108	.108	.105	.105	12.5
		100	.101	.103	.102	.101	.100	25
		300	.097	.097	.097	.096	.096	75
	2 × 4	20	.096	.167	.107	.098	.099	2.5
		50	.104	.119	.106	.102	.102	6.25
		100	.100	.106	.101	.099	.099	12.5
		300	.105	.107	.105	.104	.104	37.5
	2 × 5	20	.093	.182	.103	.102	.102	2
		50	.104	.131	.108	.104	.104	5
		100	.099	.109	.102	.099	.099	10
		300	.100	.103	.100	.100	.100	30
	4 × 5	100	.094	.136	.098	.095	.095	5
		300	.096	.104	.097	.096	.096	15

χ^2 = The Pearson χ^2 statistic
G^2 = The likelihood ratio test statistic
C-R = The Cressie and Read statistic
T = The Mielke and Berry statistic
V = The Zelterman statistic

Table 3 Empirical Type I error rates of the five statistics for moderately skewed case

Alpha	Table	N	χ^2	G^2	C-R	T	V	$\%e_{ij}<5$
.01	2 × 2	20	.011	.008	.005	.010	.007	75
		50	.009	.015	.009	.010	.010	25
		100	.009	.011	.009	.009	.009	0
		300	.010	.010	.010	.010	.010	0
	2 × 4	20	.006	.016	.005	.011	.011	87.5
		50	.007	.014	.008	.009	.009	37.5
		100	.008	.012	.008	.009	.009	0
		300	.009	.011	.010	.009	.010	0
	2 × 5	20	.004	.013	.003	.009	.009	100
		50	.007	.014	.006	.008	.007	60
		100	.007	.013	.008	.009	.009	10
		300	.010	.011	.009	.010	.010	0
	4 × 5	100	.008	.019	.007	.009	.009	65
		300	.009	.011	.009	.009	.009	0
.05	2 × 2	20	.044	.072	.048	.045	.042	75
		50	.046	.062	.049	.048	.047	25
		100	.049	.055	.052	.049	.052	0
		300	.051	.052	.050	.051	.050	0
	2 × 4	20	.044	.080	.046	.052	.051	87.5
		50	.046	.071	.050	.049	.048	37.5
		100	.052	.060	.053	.053	.053	0
		300	.048	.050	.048	.048	.048	0
	2 × 5	20	.037	.079	.036	.051	.050	100
		50	.044	.075	.046	.049	.047	60
		100	.047	.061	.047	.048	.049	10
		300	.049	.052	.049	.049	.049	0
	4 × 5	100	.047	.084	.048	.050	.049	65
		300	.046	.056	.048	.047	.047	0
.10	2 × 2	20	.097	.129	.101	.096	.089	75
		50	.100	.117	.103	.100	.099	25
		100	.100	.105	.103	.099	.099	0
		300	.106	.106	.106	.106	.104	0
	2 × 4	20	.095	.157	.099	.101	.099	87.5
		50	.099	.137	.104	.099	.100	37.5
		100	.100	.111	.102	.100	.100	0
		300	.098	.100	.098	.098	.098	0
	2 × 5	20	.095	.160	.094	.102	.104	100
		50	.093	.142	.100	.097	.097	60
		100	.096	.115	.098	.097	.097	10
		300	.098	.103	.099	.098	.098	0
	4 × 5	100	.097	.153	.100	.098	.098	65
		300	.096	.109	.098	.097	.097	0

Table 4 Empirical Type I error rates of the five statistics for extremely skewed case

Alpha	Table	N	χ^2	G^2	C-R	T	V	$\%e_{ij}<5$
.01	2 × 2	20	.020	.010	.011	.011	.010	75
		50	.012	.006	.008	.009	.008	50
		100	.010	.010	.008	.009	.008	25
		300	.009	.013	.010	.008	.009	0
	2 × 4	20	.012	.010	.006	.011	.011	87.5
		50	.010	.012	.008	.010	.009	50
		100	.010	.014	.008	.010	.010	37.5
		300	.009	.011	.009	.009	.009	0
	2 × 5	20	.012	.008	.004	.012	.010	90
		50	.011	.012	.008	.011	.009	70
		100	.010	.015	.010	.011	.011	40
		300	.010	.013	.010	.011	.011	0
.05	2 × 2	20	.057	.047	.044	.046	.035	75
		50	.041	.039	.039	.047	.028	50
		100	.040	.071	.042	.041	.040	25
		300	.048	.056	.049	.048	.051	0
	2 × 4	20	.055	.056	.044	.056	.055	87.5
		50	.044	.063	.042	.045	.042	50
		100	.047	.068	.048	.047	.048	37.5
		300	.052	.057	.053	.053	.052	0
	2 × 5	20	.051	.052	.036	.054	.049	90
		50	.048	.067	.044	.050	.049	70
		100	.049	.070	.048	.050	.050	40
		300	.050	.057	.051	.051	.051	0
.10	2 × 2	20	.076	.069	.072	.077	.102	75
		50	.075	.109	.076	.086	.073	50
		100	.090	.149	.097	.098	.113	25
		300	.101	.109	.103	.102	.106	0
	2 × 4	20	.102	.122	.095	.106	.095	87.5
		50	.087	.132	.087	.089	.091	50
		100	.098	.130	.100	.098	.100	37.5
		300	.103	.107	.103	.104	.101	0
	2 × 5	20	.105	.116	.087	.103	.101	90
		50	.094	.135	.091	.096	.096	70
		100	.100	.137	.101	.101	.101	40
		300	.097	.108	.099	.098	.099	0

Table 5 Empirical Type I error rates of the five statistics for extremely skewed case

Alpha	Case	N	χ^2	G^2	C-R	T	V	$\%e_{ij}<5$
.01	3	100	.010	.018	.008	.010	.009	65
		300	.009	.012	.008	.009	.009	10
	4	100	.012	.010	.007	.009	.009	60
		300	.011	.017	.009	.010	.010	40
	5	100	.032	.003	.010	.009	.010	80
		300	.019	.012	.011	.011	.011	55
.05	3	100	.045	.081	.043	.046	.045	65
		300	.044	.058	.044	.045	.044	10
	4	100	.051	.072	.040	.047	.046	60
		300	.046	.076	.046	.046	.046	40
	5	100	.084	.026	.040	.047	.044	80
		300	.059	.069	.047	.048	.048	55
.10	3	100	.093	.157	.094	.094	.095	65
		300	.095	.118	.095	.095	.095	10
	4	100	.097	.154	.086	.094	.094	60
		300	.092	.144	.094	.092	.095	40
	5	100	.133	.066	.080	.101	.096	80
		300	.103	.134	.087	.094	.096	55

Table 6 Statistical power for uniform distribution

Phi	Alpha	N	χ^2	C-R	T	V
.1	.01	20	.021	.021	.022	.022
		50	.030	.031	.029	.030
		100	.057	.058	.057	.056
		300	.194	.194	.194	.194
	.05	20	.073	.077	.073	.074
		50	.117	.117	.109	.109
		100	.169	.169	.167	.167
		300	.409	.410	.406	.406
	.10	20	.153	.154	.138	.144
		50	.193	.193	.188	.188
		100	.255	.256	.255	.255
		300	.528	.528	.526	.526
.25	.01	20	.079	.079	.080	.080
		50	.200	.203	.194	.195
		100	.471	.474	.471	.471
		300	.961	.961	.961	.961
	.05	20	.206	.212	.206	.207
		50	.439	.439	.419	.419
		100	.721	.722	.719	.719
		300	.992	.992	.992	.992
	.10	20	.329	.329	.307	.314
		50	.561	.561	.552	.552
		100	.804	.806	.804	.804
		300	.996	.996	.996	.996
.40	.01	20	.227	.227	.229	.229
		50	.599	.602	.593	.594
		100	.940	.941	.940	.940
		300	1	1	1	1
	.05	20	.433	.439	.433	.432
		50	.837	.837	.825	.825
		100	.986	.986	.986	.986
		300	1	1	1	1
	.10	20	.598	.598	.572	.574
		50	.904	.904	.900	.900
		100	.994	.995	.994	.994
		300	1	1	1	1

Table 7 Statistical power for moderately skewed distribution

Phi	Alpha	N	χ^2	C-R	T	V
.1	.01	20	.030	.020	.028	.023
		50	.039	.038	.041	.039
		100	.067	.061	.070	.061
		300	.210	.203	.211	.202
	.05	20	.092	.093	.095	.079
		50	.121	.121	.124	.108
		100	.184	.179	.184	.170
		300	.415	.410	.415	.402
	.10	20	.162	.157	.160	.133
		50	.203	.200	.203	.175
		100	.273	.271	.273	.251
		300	.527	.522	.527	.511
.25	.01	20	.106	.078	.106	.091
		50	.227	.220	.234	.221
		100	.472	.457	.478	.456
		300	.950	.947	.951	.947
	.05	20	.234	.231	.238	.205
		50	.433	.427	.438	.399
		100	.693	.686	.692	.671
		300	.988	.988	.988	.987
	.10	20	.340	.327	.338	.284
		50	.558	.550	.558	.502
		100	.784	.780	.784	.761
		300	.994	.994	.994	.994
.40	.01	20	.269	.218	.266	.239
		50	.604	.594	.612	.595
		100	.916	.908	.918	.908
		300	1	1	1	1
	.05	20	.472	.466	.472	.426
		50	.788	.785	.795	.760
		100	.973	.972	.973	.969
		300	1	1	1	1
	.10	20	.594	.582	.591	.526
		50	.865	.861	.866	.833
		100	.986	.986	.987	.982
		300	1	1	1	1

Table 8 Statistical power for extremely skewed distribution

Phi	Alpha	N	χ^2	C-R	T	V
.1	.01	20	.064	.038	.039	.035
		50	.062	.045	.053	.043
		100	.093	.080	.087	.077
		300	.227	.210	.224	.205
	.05	20	.138	.117	.122	.098
		50	.147	.137	.164	.108
		100	.198	.186	.200	.160
		300	.413	.397	.414	.376
	.10	20	.177	.168	.178	.175
		50	.217	.206	.237	.164
		100	.286	.271	.296	.235
		300	.524	.516	.527	.485
.25	.01	20	.174	.113	.126	.113
		50	.271	.227	.253	.219
		100	.474	.436	.453	.427
		300	.916	.908	.914	.904
	.05	20	.322	.285	.292	.243
		50	.441	.426	.464	.372
		100	.664	.645	.665	.603
		300	.971	.968	.971	.964
	.10	20	.388	.373	.388	.331
		50	.543	.529	.566	.458
		100	.750	.737	.760	.691
		300	.983	.982	.983	.978
.40	.01	20	.391	.285	.310	.289
		50	.596	.542	.571	.530
		100	.871	.848	.856	.840
		300	1	1	1	1
	.05	20	.582	.544	.551	.491
		50	.753	.740	.772	.694
		100	.942	.935	.942	.918
		300	1	1	1	1
	.10	20	.651	.639	.651	.576
		50	.825	.814	.840	.761
		100	.963	.960	.966	.946
		300	1	1	1	1

Table 9 Statistical power for uniform distribution for 2 x 5 contingency tables

C	Alpha	N	χ^2	C-R	T	V
.13	.01	20	.006	.007	.015	.015
		50	.018	.021	.023	.023
		100	.040	.043	.044	.044
		300	.174	.176	.177	.177
	.05	20	.046	.055	.066	.065
		50	.088	.094	.093	.093
		100	.140	.144	.144	.143
		300	.379	.380	.380	.380
	.10	20	.116	.131	.130	.129
		50	.165	.172	.166	.167
		100	.238	.241	.238	.237
		300	.504	.506	.504	.504
.22	.01	20	.011	.014	.027	.027
		50	.060	.066	.071	.070
		100	.193	.198	.202	.202
		300	.762	.763	.765	.765
	.05	20	.077	.089	.103	.102
		50	.200	.210	.210	.209
		100	.403	.409	.409	.408
		300	.904	.905	.905	.904
	.10	20	.170	.186	.185	.184
		50	.319	.328	.321	.320
		100	.540	.544	.540	.539
		300	.947	.947	.947	.947
.41	.01	20	.063	.073	.117	.113
		50	.465	.483	.502	.499
		100	.902	.905	.907	.907
		300	1	1	1	1
	.05	20	.255	.282	.312	.305
		50	.730	.738	.740	.738
		100	.976	.977	.976	.976
		300	1	1	1	1
	.10	20	.421	.446	.444	.434
		50	.832	.836	.832	.831
		100	.989	.989	.989	.989
		300	1	1	1	1

Table 10 Statistical power for moderately skewed distribution for 2 x 5 contingency tables

C	Alpha	N	χ^2	C-R	T	V
.14	.01	20	.007	.006	.015	.015
		50	.019	.020	.025	.024
		100	.044	.046	.047	.048
		300	.223	.226	.227	.228
	.05	20	.050	.050	.068	.066
		50	.092	.096	.099	.100
		100	.151	.156	.156	.156
		300	.451	.455	.453	.453
	.10	20	.114	.115	.127	.130
		50	.167	.175	.173	.173
		100	.251	.256	.253	.254
		300	.579	.581	.579	.580
.23	.01	20	.016	.014	.029	.029
		50	.071	.072	.085	.088
		100	.220	.219	.233	.234
		300	.806	.803	.809	.810
	.05	20	.085	.087	.108	.108
		50	.219	.223	.231	.233
		100	.434	.434	.441	.443
		300	.926	.925	.926	.925
	.10	20	.170	.169	.184	.186
		50	.332	.341	.338	.338
		100	.567	.567	.569	.569
		300	.963	.962	.963	.963
.45	.01	20	.110	.108	.185	.183
		50	.655	.668	.690	.678
		100	.983	.985	.986	.983
		300	1	1	1	1
	.05	20	.369	.373	.422	.406
		50	.873	.881	.883	.871
		100	.997	.997	.997	.997
		300	1	1	1	1
	.10	20	.533	.539	.556	.536
		50	.937	.941	.938	.929
		100	.999	.999	.999	.999
		300	1	1	1	1

Table 11 Statistical power for extremely skewed distribution for 2 x 5 contingency tables

C	Alpha	N	χ^2	C-R	T	V
.1	.01	20	.013	.005	.012	.011
		50	.020	.016	.021	.018
		100	.033	.029	.034	.031
		300	.097	.089	.097	.091
	.05	20	.065	.046	.069	.057
		50	.074	.068	.076	.070
		100	.110	.106	.111	.102
		300	.238	.233	.239	.229
	.10	20	.127	.107	.127	.114
		50	.143	.136	.146	.131
		100	.185	.183	.188	.171
		300	.350	.345	.350	.336
.28	.01	20	.041	.020	.043	.040
		50	.139	.119	.140	.131
		100	.381	.367	.385	.367
		300	.958	.959-	.958	.955
	.05	20	.152	.120	.162	.135
		50	.315	.302	.320	.294
		100	.614	.615	.617	.594
		300	.989	.990	.989	.988
	.10	20	.255	.222	.255	.216
		50	.444	.437	.449	.406
		100	.731	.734	.733	.707
		300	.995	.995	.995	.994
.42	.01	20	.101	.055	.110	.104
		50	.464	.427	.468	.456
		100	.890	.889	.888	.882
		300	1	1	1	1
	.05	20	.311	.255	.321	.286
		50	.696	.686	.699	.674
		100	.966	.969	.966	.963
		300	1	1	1	1
	.10	20	.456	.409	.457	.402
		50	.804	.804	.807	.777
		100	.984	.987	.984	.982
		300	1	1	1	1