Chapter 6
Conclusion

In this thesis we apply the many-body path integral techniques to study the system of distinguishable and indistinguishable particles. Special focus is on the phenomena of Bose-Einstein condensation in trapped gas. We study the system in both statical and dynamical aspects. In the first chapter we give a short introduction to the physics of the system and in Chapter 2 we review some necessary background in many-body path integrals. In Chapter 3 we present the variational path integral technique to study the distinguishable particles in a harmonic trap which interact with each other via a contact potential. We show that the ground state energy of the system coincides with the energy obtained from mean-field theory. It is well known that the Gross-Pitaevskii equation is the most suitable approach for such systems. However, our method is good for any form of the interaction potential because we can always find the average of the coordinates of particles and the analytical results can be obtained, unlike the Gross-Pitaevskii mean field theory which is valid for the contact potential (in the form of the Dirac delta function) only.

In Chapter 4, we study the excitation of the Bose-Einstein condensate. We find the time-evolution of the system or the time-dependent density matrix by using the real time propagator as a time-evolution generator for the imaginary-time propagator (density matrix). Firstly we aim to include interaction between particles and the statistical sum (because for our system the Bose-Einstein statistics is demanded) in the calculation. However, to do so we need the semigroup property of the time-dependent density matrix but in the interacting system we cannot find the analytic result satisfying such property. The interaction term is the cause of this difficulty. We discuss extensively about this in the last section of this chapter. Due to this fact we have to choose to retain the statistics and discard the interactions between particles. In this case we have only the system of non-interacting indistinguishable particles. We calculate the time-dependent density of the Bose system after changing the trap frequency which causes the excitation of the condensate. We found that the condensate oscillates with frequency in accordance with the experimental results. However, in the real situation this oscillation is damping with time. This is due to the interaction between the thermal cloud and the condensate.

After the work on the excitation of the normal condensate, it is a good idea to investigate the case of the condensate with vortex. In experiment the precession of the vortex was found. In Chapter 5 we simulate that situation by using the same formulation as in Chapter 4 to study the evolution of the density of the condensate with vortex. We find the precession of the vortex and the fluctuation in the density. These two effects are the collective excitations of the system. To our knowledge, this effect have not yet been investigated by any experimental group.