CHAPTER III

FINITE ELEMENT ANALYSIS

To study the influence of diagonal web reinforcement, the directions and magnitudes of stresses (strains) distributed in concrete and reinforcing steels should be known. A finite element analysis with appropriated material models can predict the responses accurately. A monotonic finite element analysis is performed in this study to approximate the envelope of the cyclic load-deformation curves. Material models used in this study are verified with the test results and then the influence of diagonal web reinforcement on shear behavior is studied.

3.1 Finite Element Formulation

The displacement-based finite element method is widely used in engineering applications. In this method, the displacement field within each element is expressed in terms of the nodal displacements by means of appropriate shape functions. The principle of virtual displacement is conveniently used to formulate the equilibrium equations. The classical principle of virtual displacement may be states as follows:

"the equilibrium of the body requires that for any compatible, small virtual displacements which satisfy the essential boundary conditions, imposed on the body, the total internal virtual work is equal to the total external virtual work" (Bathe 1982)

For a general body, the principle of virtual displacement can be written as

$$\int_V \boldsymbol{\delta\varepsilon}^{\mathrm{T}} \boldsymbol{\sigma}\, dV = \int_V \boldsymbol{\delta u}_{\mathrm{B}}^{\mathrm{T}} \mathbf{f}_{\mathrm{B}}\, dV + \int_S \boldsymbol{\delta u}_{\mathrm{S}}^{\mathrm{T}} \mathbf{f}_{\mathrm{S}}\, dS + \sum_i \boldsymbol{\delta U}_i^{\mathrm{T}} \mathbf{F}_i \qquad (3.1)$$

in which $\boldsymbol{\delta u}$ is the virtual displacement vector, $\boldsymbol{\delta\varepsilon}$ is the virtual strain vector, $\mathbf{f}_{\mathrm{B}}, \mathbf{f}_{\mathrm{S}}, \mathbf{F}_i$ are vectors of body forces, surface tractions, and concentrated forces, respectively, $\boldsymbol{\delta u}_{\mathrm{B}}, \boldsymbol{\delta u}_{\mathrm{S}}, \boldsymbol{\delta U}_i$ are, respectively, vectors of virtual displacements associated with $\mathbf{f}_{\mathrm{B}}, \mathbf{f}_{\mathrm{S}}, \mathbf{F}_i$, and $\boldsymbol{\sigma}$ is a vector of actual stresses.

In the finite element analysis, the displacements within each element are assumed to be a function of the displacements at the finite element nodal points.

$$\mathbf{u}^{(m)}(x,y) = \mathbf{H}^{(m)}(x,y)\mathbf{U} \tag{3.2}$$

where $\mathbf{u}^{(m)}$ denotes the continuous displacement field within the element m, $\mathbf{H}^{(m)}$ is an appropriate displacement interpolation matrix, and $\mathbf{U}$ is the vector of global nodal displacements.

The corresponding element strains, $\varepsilon^{(m)}$, can now be related to the nodal displacements through the strain-displacement matrix, $\mathbf{B}^{(m)}$, as follows:

$$\varepsilon^{(m)}(x,y) = \mathbf{B}^{(m)}(x,y)\mathbf{U} \tag{3.3}$$

The stresses in the element are then related to element strains through the constitutive matrix, $\mathbf{C}^{(m)}$. Thus,

$$\sigma^{(m)} = \mathbf{C}^{(m)}\varepsilon^{(m)} \tag{3.4}$$

By substituting Eq. (3.2) through Eq. (3.4) for both real and virtual displacements and strains into equation (3.1) and rearranging the terms, the principle of virtual displacement can be rewritten as follows:

$$\mathbf{KU} = \mathbf{R} \tag{3.5}$$

where $\mathbf{K}$ is the stiffness matrix obtained from the element assemblage, and the load vector $\mathbf{R}$ includes the effect of the element body forces, $\mathbf{R}_b$, the effect of the element surface forces, $\mathbf{R}_s$, and the effect of concentrated loads, $\mathbf{R}_c$, which are, respectively, given by

$$\mathbf{K} = \sum_m \int_{V^{(m)}} \mathbf{B}^{(m)^T}\mathbf{C}^{(m)}\mathbf{B}^{(m)} dV^{(m)} \tag{3.6}$$

$$\mathbf{R}_B = \sum_m \int_{V^{(m)}} \mathbf{H}^{(m)^T}\mathbf{f}_B^{(m)} dV^{(m)} \tag{3.7}$$

$$\mathbf{R}_S = \sum_m \int_{S^{(m)}} \mathbf{H}^{(m)^T}\mathbf{f}_S^{(m)} dS^{(m)} \tag{3.8}$$

$$\mathbf{R}_C = \mathbf{F} \tag{3.9}$$

3.2 Iterative Method

Equation (3.5) is valid for linear systems with constant structural stiffness. This constant stiffness matrix results from the use of a constant constitutive matrix, $\mathbf{C}$, and a constant strain-displacement matrix, $\mathbf{B}$, which are valid for a linear elastic structure undergoing small displacement. As a result, the nodal displacements, $\mathbf{U}$, corresponding to the applied load vector, $\mathbf{R}$, can be calculated directly from Eq. (3.5). However, the stress-strain relationships of concrete and reinforcing steel are nonlinear and depend on the current state of stresses and strains. This will cause the constitutive matrix and, consequently, the stiffness matrix to be nonlinear. Therefore, an incremental-iterative algorithm is required to solve the nonlinear equilibrium equations.

The state of equilibrium for the nonlinear structure can be obtained following the procedure shown in Fig. 3.1. The displacements are first calculated by Eq. (3.5) with initial stiffness of the structure. Then, the stresses due to the calculated deformations of each element are obtained from Eqs. (3.3-3.4). Consequently, the nodal forces in equilibrium with the element stresses can be evaluated by Eq. (3.10).

$$\mathbf{F}_i = \sum_m \int_{V^{(m)}} \mathbf{B}^{(m)^T} \boldsymbol{\sigma}_i^{(m)} \, dV \qquad (3.10)$$

These forces will be compared with the applied forces, $\mathbf{R}$. The system will be in equilibrium if the Euclidean norm of the unbalanced forces, which is the difference between calculated forces, $\mathbf{F}_i$, and the applied forces, $\mathbf{R}$, ($\Delta\mathbf{F}_i = \mathbf{R} - \mathbf{F}_i$), is less than the acceptable tolerance (taken as 5% of the norm of applied forces in this study). Otherwise, the equilibrium of the system is not satisfied. The structural stiffness will then be updated using the constitutive matrix obtained from the secant stiffnesses of the materials at the current state of stresses and strains. The incremental displacements due to the unbalanced forces will then be estimated from Eq. (3.11).

$$\mathbf{K}_{i+1} \Delta\mathbf{U}_{i+1} = \Delta\mathbf{F}_i \qquad (3.11)$$

The updated displacement can be obtained from the summation of the previous displacement and the incremental displacement ($\mathbf{U}_{i+1} = \mathbf{U}_i + \Delta\mathbf{U}_{i+1}$). The iteration is performed until the equilibrium is reached. The details of the described procedure, known as direct iteration using the secant stiffness of the structure, have been given by Zienkiewicz and Taylor (1991).

3.3 Finite Element Model

3.3.1 Modeling for reinforcing steel

The major models of steel reinforcement which have been used successfully in the finite element analysis of reinforced concrete are

1. discrete steel model
2. smeared steel model

In a discrete model, a reinforcing bar is represented by a one-dimensional bar element. The advantages of this model are its simplicity and its ability to include bond-slip relationships between concrete and steel by using a linkage element to connect the common nodes of a bar element and concrete element. The disadvantage of the discrete model is its mesh discrepancy; the direction and location of bar elements depend on the mesh layout of the finite element model.

In a smeared model, reinforcing steel is assumed to be uniformly distributed over a concrete element. The perfect bond between reinforcing steel and concrete must also be assumed in this model. The constitutive matrix of reinforcing steel is superimposed on top of the constitutive matrix of concrete to obtain the total constitutive matrix of reinforced concrete.

3.3.2 Modeling for concrete

The stress and strain are assumed to be continuous within each element in the finite element formulation. However, when concrete cracks, discontinuities in stress and strain occur. In general, the approaches which have been used to represent cracks are

1. discrete crack model
2. smeared crack model

In the discrete crack model, cracks are represented as a separation of nodes along element boundaries. The post-cracking behavior such as aggregate interlock, dowel action, and bond slippage can be incorporated into the model by using linkage elements to connect the separated nodes. Many finite element models have yielded smaller displacements than the experimental results when these influences are overlooked (Okamura and Maekawa 1991). Okamura and Maekawa (1991) modeled these characteristics at joint between shear wall and footing by using a one-dimensional element with the relationships between force and displacement.

In the smeared crack model, concrete is assumed to remain continuous after cracking. The stress-strain discontinuities across the cracks are averaged over the element in the vicinity of the cracks; consequently, the stress-strain relationship of cracked concrete can still be described in a continuous manner. There are two major crack models generally used (Kwan and Billington 2001):

1. fixed crack model
2. rotating crack model

For the fixed crack model, cracks occur normal to the direction of the maximum principal stress when the maximum principal stress first reaches the cracking stress. This crack direction is assumed to remain fixed throughout the analysis. Then the cracked concrete is assumed to be orthotropic, with respect to the axes that are parallel and normal to the first crack direction. The constitutive relationship of cracked concrete for two-dimensional plane stress problems can be written in the crack coordinates (See Fig. 3.2) as follows:

$$\mathbf{C}^c = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & G \end{bmatrix} \tag{3.12}$$

in which E_1 and E_2 are the stiffnesses of cracked concrete normal and parallel to the first crack, respectively. As pointed out in the Section 3.2 "Iterative method", the secant stiffness is employed in this study. Therefore, E_1 and E_2 are determined from the current states of stresses σ_1 and σ_2 associated with the strains ε_1 and ε_2, respectively. Thus,

$$\begin{aligned} E_1 &= \frac{\sigma_1}{\varepsilon_1} \\ E_2 &= \frac{\sigma_2}{\varepsilon_2} \end{aligned} \tag{3.13}$$

The shear modulus, G, in the fixed crack model represents the shear stiffness retained in the crack direction because of aggregate interlock and dowel action. At first, the cracked shear modulus is obtained by multiplying the elastic uncracked shear modulus by an empirical shear retention factor ranging from 0 to 1. It was found that any nonzero value of the factor was satisfactory as results were relatively insensitive to the value assumed (Grayson and Stevens 1979). Therefore, a number of researchers proposed empirical shear functions that satisfy their problems. The summary of these functions can be found in the paper by Zsu et al. (2001). Because of the complicated forms of shear stress-strain relationship of cracked concrete, Zsu et al. (2001) has proposed a rational shear modulus that is derived from the equilibrium and compatibility conditions. Figure 3.2 depicts a stress field in global (x-y axes), principal (r-d axes), and crack (1-2 axes) directions. The equilibrium and compatibility equations between the crack and principal directions are shown in Eq. (3.14) and Eq. (3.15), respectively.

Equilibrium equations:

$$\begin{aligned} \sigma_1 &= \sigma_d \sin^2\beta + \sigma_r \cos^2\beta \\ \sigma_2 &= \sigma_d \cos^2\beta + \sigma_r \sin^2\beta \\ \tau_{12} &= (\sigma_r - \sigma_d)\sin\beta\cos\beta \end{aligned} \tag{3.14}$$

Compatibility equations:

$$\begin{aligned} \varepsilon_1 &= \varepsilon_d \sin^2 \beta + \varepsilon_r \cos^2 \beta \\ \varepsilon_2 &= \varepsilon_d \cos^2 \beta + \varepsilon_r \sin^2 \beta \\ \gamma_{12} &= 2(\varepsilon_r - \varepsilon_d)\sin\beta\cos\beta \end{aligned} \tag{3.15}$$

Therefore, the shear modulus along the crack is given by

$$\begin{aligned} G = \frac{\tau_{12}}{\gamma_{12}} &= \frac{(\sigma_r - \sigma_d)}{2(\varepsilon_r - \varepsilon_d)} \\ &= \frac{(\sigma_1 - \sigma_2)}{2(\varepsilon_1 - \varepsilon_2)} \end{aligned} \tag{3.16}$$

For the rotating crack model, the cracked concrete is assumed to be orthotropic, as it is assumed in the fixed crack model. However, the axes of orthotropy do not remain fixed but they are always aligned with the principal directions.

3.4 Material Model for Reinforcing Steel

The stress-strain curve for reinforcing steel in tension is shown in Fig. 3.3. It consists of four linear segments representing the initial elastic portion, the yield plateau, strain hardening, and tensile capacity plateau. Figure 3.3 also compares the stress-strain relationship from the tensile test to that represented by the model.

The stress-strain relationship in compression is not the same as in tension because steel bars under high compression tend to buckle and it is necessary to model buckling of reinforcing steel. The buckling behavior depends on the slenderness of reinforcing bar which is defined by the ratio of buckling length, L, to diameter of reinforcing bar, D, the yield strength, f_y, and the lateral restraint provided by transverse ties (Dhakal and Maekawa 2002).

It should be noted that the buckling length of reinforcing bars inside an RC member is generally not equal to the spacing of lateral ties, except in special cases when the lateral ties are very stiff, or the reinforcing bars are very slender, or the spacing is very large. Therefore, Dhakal and Maekawa (2002) has proposed a simple method to predict the buckling length. First, the actual stiffness of the lateral ties effective to each bar is computed. Next, the minimum ties stiffnesses required to hold a vertical bar in different buckling modes are determined by means of energy principles. The buckling mode n is defined as buckling over n tie spacing which is the buckling length. If the actual tie stiffness is less than the required stiffness for mode n-1 but exceeds that for mode n, the vertical bar would buckle in the n^{th} buckling mode. Table 3.1 summarizes the stiffness required to stabilize the reinforcing bar in each buckling mode.

For the longitudinal bars in the boundary elements, the lateral ties stiffness, k_t, is given by Eq. (3.17).

$$k_t = \frac{E_t A_t}{l_e} \times \frac{n_l}{n_b} \tag{3.17}$$

where E_t is Young's modulus of lateral tie, A_t is cross-sectional area of lateral tie, l_e is effective length of tie, n_l is the number of lateral tie legs along the buckling direction, and n_b is the number of main bars prone to simultaneous buckling.

For the walls tested, k_t is found to be 54 kN/mm, and according to Table 3.1, this stiffness value can hold the bars in the second buckling mode. Therefore the buckling length of the longitudinal bars is 2x60 = 120 mm which was actually confirmed by the experiment.

For the web reinforcement, there is no restraining lateral tie and the procedure described above cannot be applied. Instead, the buckling length of web bars is assumed to be equal to the spacing of web reinforcement in view of its large spacing. Furthermore, the web bars in tension provides some lateral restraints for the bars in compression.

Dhakal and Maekawa (2002) has also proposed the stress-strain relationship of reinforcing bars including buckling effect based on analytical parametric studies using the fiber technique. The model assumes that a bar begins to be softened after yield point. The softening in the average compressive stress-strain relationship has been found to be related by $\frac{L}{D}\sqrt{f_y}$. Regardless of $\frac{L}{D}\sqrt{f_y}$, the compressive stress degradation rate in the second state beyond an intermediate state $(\varepsilon^*, \sigma^*)$ is nearly constant with a negative slope approximately equal to 2% of elastic modulus, $0.02\,E_s$, until the average compressive stress reaches 20% of the yield strength, $0.20\,f_y$ after which the stress becomes constant. The complete compressive stress–strain relationships in monotonic loading are given by

$$\begin{aligned} \sigma_s &= E_s \varepsilon_s && ;\varepsilon \le \varepsilon_y \\ \frac{\sigma_s}{\sigma_l} &= 1.0 - \left(1 - \frac{\sigma^*}{\sigma_l^*}\right)\left(\frac{\varepsilon - \varepsilon_y}{\varepsilon^* - \varepsilon_y}\right) && ;\varepsilon_y < \varepsilon \le \varepsilon^* \\ \sigma_s &= \sigma^* - 0.02E_s\left(\varepsilon - \varepsilon^*\right) && ;\varepsilon > \varepsilon^* \\ &\ge 0.2 f_y \end{aligned} \qquad (3.18)$$

where (ε,σ) is a current point, (ε^*,σ^*) is an intermediate point, σ_l and σ^*_l are the stresses in the tension envelope corresponding to ε and ε^*, respectively. The intermediate point (ε^*,σ^*) is defined as follows:

$$\begin{aligned} \frac{\varepsilon^*}{\varepsilon_y} &= 55 - 2.3\frac{L}{D}\sqrt{\frac{f_y}{100}} \\ \frac{\sigma^*}{\sigma_l^*} &= 1.1 - 0.016\frac{L}{D}\sqrt{\frac{f_y}{100}} \end{aligned} \qquad (3.19)$$

where (ε_y, f_y) is the yield point, σ^*_l are the stresses in the tension envelope corresponding to ε^*.

Figure 3.4 shows the stress-strain relationship of the longitudinal bars and web bars following the Dhakal and Maekawa (2002) model, and the relevant properties are given in Table 3.2. Table 3.3 summarizes the buckling strengths of the web bars computed in accordance with Dhakal and Maekawa, together with the elastic buckling values. It is seen that the elastic buckling strengths of the web bars of specimens WC150 and WD150 are higher than the yield strengths while those of the other specimens are less than the yield values. This indicates that the web bars in specimens WD170, WD200 and WCD170 should have buckled elastically at the lower value of the buckling load, and the Dhakal and Maekawa inelastic buckling model is not applicable. Therefore, Eq. (3.20) is proposed to model bars that buckle elastically. Figure 3.5 compares the stress-strain relationships of web bars using this proposed model and Dhakal and Maekawa model.

$$\begin{aligned}
\sigma &= E_s\varepsilon && ;\varepsilon_b < \varepsilon \leq \varepsilon^{*} \\
&= \sigma_b - \left(\frac{\sigma^{*} - f_y}{\varepsilon^{*} - \varepsilon_y}\right)(\varepsilon - \varepsilon_b) && ;\varepsilon > \varepsilon^{*} \\
&\geq 0.2 f_y
\end{aligned} \tag{3.20}$$

where ε_b and σ_b is the elastic buckling strain and stress, respectively.

3.5 Material Model for Concrete

The constitutive modeling of smear fixed cracked concrete requires the stress-strain relationship of concrete in the crack direction. Since the stress-strain relationship of concrete is generally determined by uniaxial testing, it is convenient and reasonable to modify and apply the uniaxial stress-strain relationship of concrete in the cracked direction. The concrete stress-strain relationships proposed by Hognestad (1951) and Saatcioglu and Razvi (1992) were adopted for unconfined and confined concrete, respectively (see Fig. 3.6). Tension in concrete was modeled using the formulation proposed by Belarbi et al. (1994) (see Fig. 3.7).

Concrete has been customarily modeled either as unconfined or confined. For concrete web, it is reasonable to model it as unconfined since no transverse reinforcement is provided. The state of stress in the boundary element with sufficient confining transverse reinforcement is much more complex. When buckling of the longitudinal reinforcement takes place, the confinement pressure on the concrete core adjacent to the buckled bars decreases, i.e., the confinement effectiveness is reduced after buckling of the longitudinal bars. As mentioned in the previous section, the buckling length of the longitudinal reinforcement in the columns considered in this study equals twice the spacing of the lateral ties. Fig. 3.8 depicts the confined concrete behavior at different levels of confinement. With the confining stress effectively provided by all lateral ties, the confined concrete stress-strain model exhibits a slow decay in the descending branch as illustrated by line 1 in the figure. If the lateral tie spacing is doubled, the effective confinement is reduced by half, resulting in the concrete stress-strain relationship as shown by Line 2 in Fig. 3.8. Therefore, the stress-strain relationship of confined concrete during buckling should be somewhere between Line 1 and Line 2. As a simplified approximation, only the descending branch of the confined stress-strain model is modified accounting for the buckling effect, which is represented by Line 3, reducing linearly from the peak confined stress to the average stress of Line 1 and Line 2 at the intermediate strain of the longitudinal bar (ε^*).

Shear walls are actually 3D structures. However, modeling the walls using 2D finite element idealization in the x-y plane is simple and widely adopted. This implies that the same state of stress is assumed to exist throughout the thickness of the wall. To capture the 3D behavior, realistic modeling of the actual stress variations in the unconfined concrete cover and the confined concrete core is needed.

Assuming the same state of strain exists across the entire depth of the wall (perpendicular to the lateral loading direction), the actual virtual strain energy for any imposed virtual strain may be equated to the virtual strain energy performed by the equivalent effective stress which is assumed to be constant throughout the thickness of the wall. Thus,

$$\begin{aligned}\int_V \delta\varepsilon^T \sigma\, dV &= \int_V \delta\varepsilon^T \sigma t\, dxdy \\ &= \int_V \delta\varepsilon^T \left[\sigma_{uc}\left(t - t_c\right) + \sigma_c t_c\right] dxdy\end{aligned} \tag{3.21}$$

where σ is the effective equivalent stress, σ_{uc} and σ_c are, respectively, the unconfined and confined stresses corresponding to the same level of strain, ε . t_c is the thickness of core concrete measured center-to-center of the hoop and t is the total thickness of boundary columns.

Therefore, the effective equivalent stress, σ, can be obtained by Eq. (3.22) which depends on the ratio of the thickness of core concrete to the total thickness of boundary column.

$$\sigma(\varepsilon)=\left(1-\frac{t_c}{t}\right)\sigma_{uc}(\varepsilon)+\left(\frac{t_c}{t}\right)\sigma_c(\varepsilon) \quad (3.22)$$

Line 3 in Fig. 3.9 represents the effective equivalent stress-strain relationship for concrete in the boundary elements, which is determined from the unconfined concrete model (Line 1) and confined concrete model including the buckling effect (Line 2) using Eq. (3.22). It is seen that there are two segments in the descending branch. To further simplify the model, one single linear line connecting the peak point and the point at intermediate strain (ε^*) is used instead.

The material properties of unconfined concrete used for concrete web elements and modified-confined concrete considering reduced confinement by buckling of longitudinal bars and combination of unconfined and unconfined material properties used for concrete boundary elements are summarized in Table 3.4.

3.6 Verification of Material Models

Figure 3.10 depicts the finite element model of the walls used in this study. Concrete was modeled using four-node bilinear isoparametric elements with a smeared, fixed crack model, and the shear modulus along crack proposed by Zhu et al. (2001) is adopted. Reinforcing bars are modeled as truss elements. Perfect bond between concrete and steel is assumed. Vertical load is applied directly in the form of prescribed loads and, in order to investigate the behavior of the structures under lateral loads, the horizontal loads are applied indirectly through prescribed displacements at the loading points.

The material models described in the previous section were incorporated into the finite element program, FINITE. To verify the material models, all specimens were analyzed and the results were compared with the experimental results. The criteria to determine a failure are described below:

a. Web crushing failure was assumed to occur when the compressive strains at all gauss points of the concrete web elements along the direction of the cracked coordinate reached the limiting strain of unconfined concrete (0.0038).

b. The buckling of longitudinal bars in the boundary elements was assumed to take place when the compressive strains reached ε^{*}, as defined in Fig. 3.6. At this strain level, the confinement provided by the transverse reinforcement was deemed to be ineffective and crushing of the concrete in the boundary elements was expected.

c. As previously described, the diagonal web bars experience compressive strains. Buckling of the web reinforcement was assumed when the compressive strains reached the buckling strain, with subsequent spalling of the concrete cover in the web.

d. Fracture of longitudinal bars due to low-cycle fatigue was not considered in this study.

Figure 3.11 compares the load-displacement relationships from the finite element analysis with those from the experiments. The experimental displacement plotted in Fig. 3.11 is the total displacement subtracted by the sliding component because sliding at base is not considered in the finite element model. The load-displacement curves predicted by the finite element analyses are seem to match reasonably well with the envelopes of the experimental hysteretic loops. The important behavior captured by the finite element analysis is compared to that obtained from experiments as listed below.

Specimens	Finite element analyses	Experiments
WC150	Maximum load is 855 kN. Web concrete (elem. 76) starts crushing at 35.4 mm.	Maximum load was 908 kN. The specimen failed by web crushing at 40.5 mm.
WD150	Maximum load is 970 kN. Buckling of longitudinal bar (elem. 234) is observed at 48.1 mm.	Maximum load was 925 kN. Buckling of longitudinal bars was observed at 35.3 mm.
WD170	Maximum load is 926 kN. Buckling of a web bar (elem. 358) and longitudinal bar (elem. 234) are predicted at 34.9 mm and 44.1 mm, respectively. Web concrete	Maximum load was 996.5 kN. Buckling of a web bar was observed at 33.0 mm. Longitudinal bars buckled at 40.7 mm.

	(elem. 76) crushing and buckling of web bar (elem. 374) then follow at 46.7 mm.	
WD200	Maximum load is 891 kN. Buckling of web bars is predicted at 38.0 mm (elem. 358), 42.5 mm (elem. 374) and buckling of longitudinal bar is predicted at 45.6 mm. Web concrete (elem. 76) crushing then follows at 50.2 mm.	Maximum load was 928 kN. Web bars buckled at 31.2 mm and web concrete crushed at 42.7 mm.
WCD170	Maximum load is 927 kN. Buckling of web bar (elem. 358) and longitudinal bar (elem. 234) are predicted at 38.0 mm and 45.1 mm, respectively. Web concrete (elem. 76) crushing then follows at 49.0 mm.	Maximum load was 914 kN. Concrete cover spalled off at 40.0 mm. and longitudinal bars buckled and fractured at 45.0 mm
WD170A	Maximum load is 1054 kN. Web bar (elem. 358) buckles at 26.8 mm. Web concrete (elem. 76) crushing and buckling of a longitudinal bar (elem. 234) are predicted at 34.4 mm.	Maximum load was 1055 kN. Concrete cover spalled off at 33.5 mm. Consequently, longitudinal bars buckled and fractured at 42.0 mm.

3.7 Influence of Diagonal Web Reinforcement

The experimental results appear to indicate that web crushing, which causes a very brittle mode of failure, is prevented when the web reinforcement is oriented in the diagonal directions. Unfortunately, due to the large variations in the compressive strength of the concrete used to construct the test specimens, it is not possible to make a definite conclusion regarding this hypothesis. To gain additional insight into the influence of diagonal reinforcement on the lateral response of walls, finite element analyses were conducted for all specimens using the same material properties of

WC150. The specimens will now be denoted by WD150*, WD170*, WD200*, WCD170*, WD170A* which are identical to WD150, WD170, WD200, WCD170, WD170A, respectively, except for the material properties as mentioned.

Figure 3.12 plots the load-displacement curves obtained from the finite element analyses. It is seen that the walls with diagonal web reinforcement have higher ductility capacity. The specimen WC150 fails by web crushing while diagonal walls fail by buckling of longitudinal bars except specimen WD200*. Because the buckling strength of web reinforcement in specimen WD200* is very low, the specimen fails by bar buckling of diagonal web reinforcement at a lower ductility. It is seen that the maximum drift attained by specimen WD150* is higher than specimen WC150 by about 22%. For specimens WD170* and WCD170*, the drift capacity is almost equal although the elastic stiffness of specimen WD170* is slightly higher than that of specimen WCD170*. Therefore, this indicates that the combination type of web reinforcement proposed appears to be quite promising for practice.

The axial load influences stiffness, load capacity, and ductility. The specimen WD170A* fails by buckling of longitudinal bar at a smaller displacement compared with specimen WD170* by about 20%. In other words, ductility decreases with an increase in the axial load. However, the results from experiments did not exhibit this behavior and further research is needed to study the wall performance under different levels of axial load.

Figure 3.13 plots the horizontal reaction force distribution at the base of specimen WC150 in which the shear force is resisted only by concrete struts, i.e., the contribution of dowel action in the vertical steel is neglected. At a small lateral displacement, the shear force is resisted mostly by the boundary column (Nodes 11-13). At the ultimate drift ratio of 1.6%, the compressive strain in the column reaches the peak confined strain. Consequently, the shear resistance by the column decreases and thus the remaining shear has to be transferred to the web concrete, thereby significantly increasing the diagonal compressive strain in the concrete strut. This explains why web crushing failure follows after excessive deformation takes place the boundary element. For wall specimen WD150*, on the other hand, part of the shear force is resisted by diagonal web reinforcement, the contribution being 8.6%, 13.0%, and 18.5% of the lateral loads at drift ratio of 0.9%, 1.6%, and 2.0%, respectively, as shown in Fig. 3.14. Consequently, the diagonal compressive strain in concrete strut is reduced.

Figure 3.15 shows the diagonal compressive strain in the concrete strut of element 76 which is the critical element in the web concrete. It confirms that with diagonal web reinforcement, the compressive strain in the web concrete strut is reduced by about 23% at the drift ratio of 1.6% (the ultimate value of WC150). Therefore, walls with diagonal web reinforcement are less susceptible to web crushing.

3.8 Strain and Stress Distributions

To develop a simplified design procedure for walls considering web crushing, the stress and strain distributions in the wall should be clearly depicted. Therefore, the distributions of strains and stresses through the wall section at the base obtained from the finite element analyses are investigated in detail in this section.

Figures 3.16-3.17 plot the strain and stress distributions in global coordinates (x-y axes) of concrete elements at the section of the base of specimen WC150 at the peak load. With regard to the strain distributions (see Figs. 3.16), the shear strains can be seen to be nearly constant and the transverse strains in x direction are close to zero. As expected, the longitudinal strains do not vary linearly especially in the region of high tensile strains. However, the assumption of linear distribution of longitudinal strains is generally acceptable in sectional analysis. The assumptions on the strain distributions described here will be used in the simplified design procedure for walls considering web crushing that will be described in the next chapter.

3.9 Summary

In this study, the finite element procedure proposed by Sittipunt (1994) is extended to predict the envelope curve of the cyclic hysteresis loops obtained from experiments, taking into account the effects of buckling of longitudinal bars on the behavior of confined concrete and the difference in stress-strain characteristics of the cover and core concrete in the boundary columns. Finite element analyses indicate the effectiveness of diagonal web reinforcement in reducing the compressive strain in the critical strut in comparison with the conventional one. The reduction is about 23% at the ultimate drift ratio of the latter, thereby deferring web crushing with enhanced performance. Consequently, the drift capacity increases by 22%.