CHAPTER 4

MODEL FORMULATION

4.1 Introduction

Due to the competitive demand of different models of product, the company needs to produce different product models to attract different customer groups. This leads the company to cooperate with the variation in order to produce at the desired output rate. This rate is related to demand forecasts that the production rates should match the demand rates as closely as possible. Matching ensures on-time delivery and prevents the buildup of unwanted inventory. The output rate also depends on rebalancing frequency, capacity utilization, and job specialization. Each time a line is rebalanced, the jobs of many operators must be redesigned. Also, the balancing concept is used when setting up the new product line. The changeover may even require a new layout, new machines, new equipment, etc. Moreover, there is the competitive advantage if the company can spend less time for designing its balancing lines and early launch new product to the market, especially in the hard disk drive industry. Therefore, constructing a mathematical model which provides productivity line, optimal output and flexibility to change is needed in this study.

4.2 Overview

First, the concept of the model is to generate the maximum output by using the minimum resource such as manpower, tooling as shown in Figure 4.1 below.

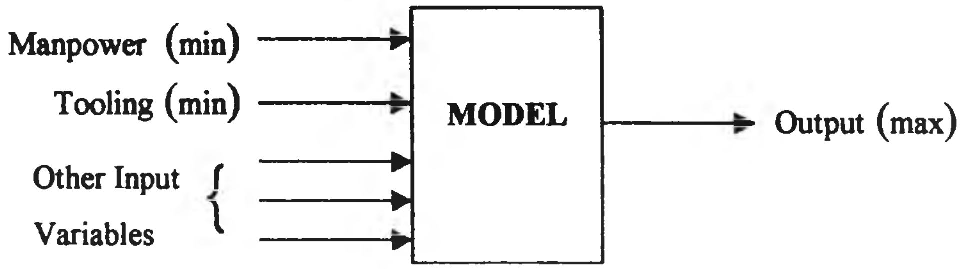

Figure 4.1: Concept of the Model

4.2.1 Input Variable Consideration

The fishbone diagram as illustrated in Figure 4.2 indicates the factors, which affect the assembly line and need to concern as the input variable in the model.

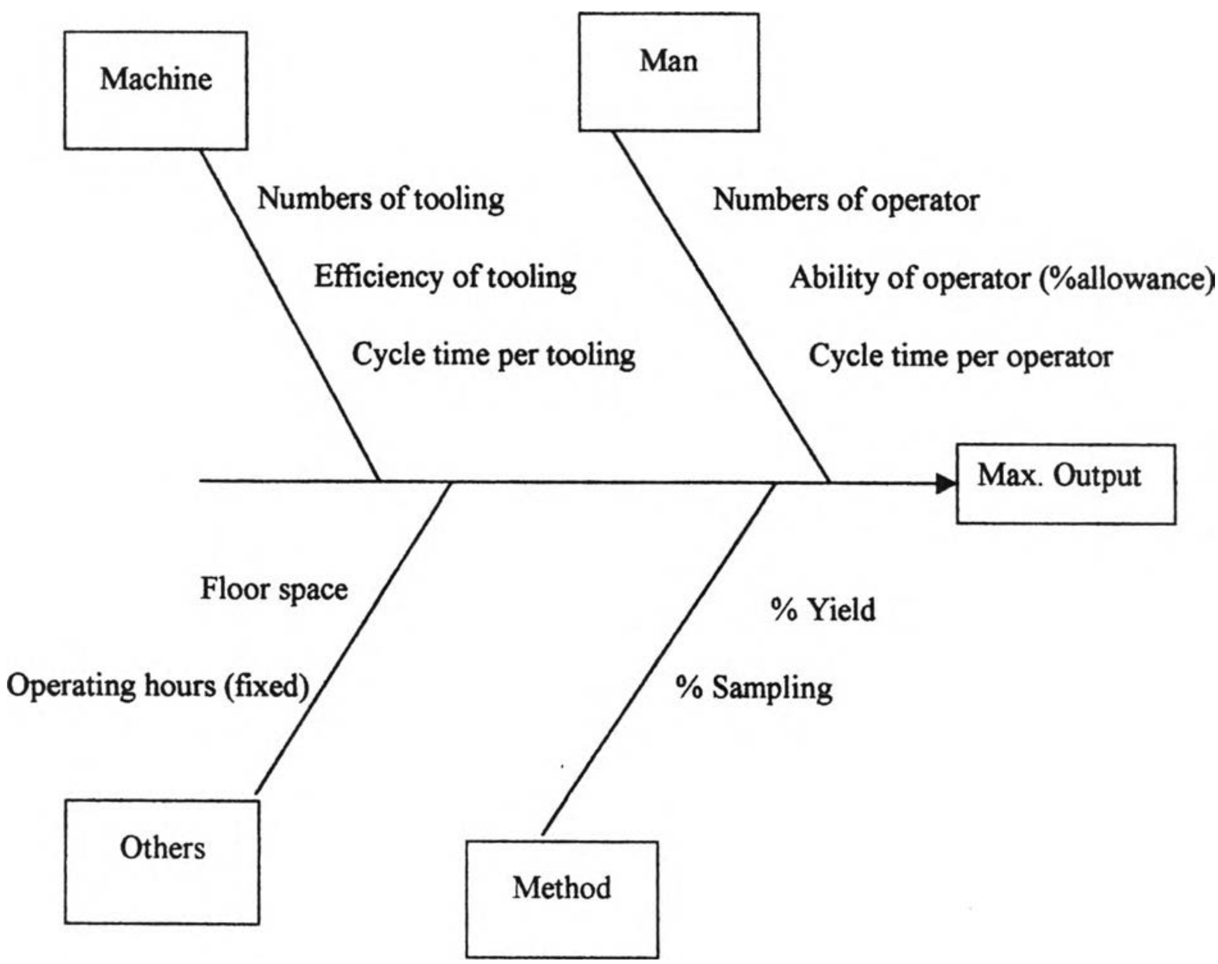

Figure 4.2 : The fishbone diagram of input variables

Man

- **Numbers of operator**

When producing the maximum output, the variables that needed to concern first is the numbers of operator needed at each operation. This number may be restricted by the numbers of operator available at each operation because some operations may require high-skill operators that the company owns limit.

- **The ability of operator**

It is the fact that the operators can not do their job extremely full speeds all the working time. There should be the percentage of allowance for such thing as fatigue, personality and others.

- **Cycle time per operator**

The time that each operator needs to perform his/her job in each operation. This cycle time is measured in seconds per one unit finished at each operation, or measured as the time when the unit enters until it ends at each operation.

Machine

- **Numbers of tooling**

This indicates the numbers of tooling available for each operation. In this study, tooling can be classified in two types: fixture and machine. Fixture is concerned as inexpensive tooling and easy to provide, so the company can own large amount of this fixture. Unlike the fixture, machine is concerned as an expensive tooling, hard to provide and limited.

- **The efficiency of tooling**

The efficiency of the tooling when operating needed to be concern. If the problem of the tooling, such as downtime, is small; the efficiency is set as 95%. If the problem is slightly bigger, this factor is set as 90%, for example.

- **Cycle time per tooling**

Like the cycle time per operator, cycle time per tooling is the time that each tooling uses to perform the job at each operation. However, in this case study, it may be no cycle time for some tooling. For example, fixture is only the equipment for operator to assembly the product, it does not require time to perform the job. However, differently from fixture, some tooling has cycle time, such as the tester machine requires time to test the product. Therefore, this factor needs to be concerned and it is appropriate to use in calculation if this factor exists.

Method

- **% Yield**

This factor means the percentage of good product produced or the percentage of good output per input. Usually, there is the case that the percentage of yield is different between each operation.

- **% sampling**

If there is the sampling or inspection in the operation, this percentage is used. Otherwise, this factor is set as 100%

Others

- **Floor space**

Floor space should be used for the most usability as possible. Floor space is restricted, so this factor concerns the space available to assign tooling and operators in order to perform task in a line. In this case study, the most usable line is restricted for the 54 stations as presented above.

- **Operating hours**

This number is fixed and depends on the policy of each company, for example, operating hours is 8 hours per day. In this case study, the operating hours is 21 hours per day.

4.3 Constructing a Mathematical Model

After receive the objective of the model and the variables that affect the model, we can construct the model by using the mathematics to present the relationship of the input variables. Then, the theory of mathematical model is used.

Concept Objective Function: Maximize **Final Output**

Constraints: Operators

Tooling

Space

Assumptions

1. An operator performs only one work element at a time, and the operators perform the same work element in the same manner in a same operation.
2. Cycle time measured in each operation is an average value of actual cycle time and is considered to be constant.

Design variables

i $\equiv$ number of operations ($i = 1, \ldots\ldots\ldots, n$)

- **Decision variables**

$\mathbf{x}_i$ $\equiv$ numbers of operator at operation i

$\mathbf{m}_i$ $\equiv$ numbers of tooling at operation i

- **Independent variables**

$\mathbf{a}_i$ $\equiv$ cycle time per operator at operation i

$\mathbf{b}_i$ $\equiv$ cycle time per tooling at operation i

$\mathbf{T}_i$ $\equiv$ total operating time at operation i

$\mathbf{s}_i^o$ $\equiv$ space factor of each operator at operation i

s_i^m ≡ space factor of each tooling at operation i

- **Dependent variables**

d_i ≡ output at operation i

s_i ≡ space factor at operation i

MODEL

This model is formulated as the generic model for any assembly line. Moreover, it is the integer programming model, since the variables are integer variables. It means that the variables cannot be the fraction values such as number of 2.5 operators. And it assumes variables can be any non-negative fractional number. Here are the presentations and the explanations of the constraints in the model.

Objective Function : Maximize **Final Output (d_n)**

S.T.

1. The continuity of output at each operation

$$d_{i+1} \leq d_i \qquad i = 1,2,3,\ldots\ldots,n-1 \qquad \ldots\ldots\ldots(4.1)$$

In this assembly line, the process is the output of one operation is sent to be an input for the next operation, until the end of the line. Thus, the numbers of output at one operation depend on the numbers of output at its previous operation. For example, if the numbers of output at Operation 1 is 100 items. The numbers of output at Operation 2 can not be more than 100 items, even though it has capability to produce more than 100 items. On the contrary, if the capability of Operation 2 is less than 100 items, ex. 80 items, so the numbers of output at Operation 2 are 80 items. Therefore, the numbers of output at one operation must less than or equal to the numbers of output at previous operation.

2. The relationship of the output with the operator and tooling

$$d_i = f(m_i , x_i)$$

The numbers of output at each operation depend on the numbers of operator and the numbers of tooling used in each operation.

$$\text{Output at operation i} = \frac{\text{total operating time}_i}{\text{cycle time}_i} \times \text{(numbers of operator or tooling)}$$

There are three cases of this function:

<u>Case1</u> If $\mathbf{m}_i = 0$ (no tooling or Fully Manual Operation)

$$\mathbf{d}_i = \frac{\mathbf{T}_i}{\mathbf{a}_i} \times \left(\frac{\mathbf{\%Yield} \times \mathbf{\%Efficiency}}{\mathbf{\%Sampling}}\right)_i \times \mathbf{x}_i \qquad \text{.....................(4.2)}$$

<u>Case2</u> If $\mathbf{x}_i = 0$ (no operator or Fully Automatic Line)

$$\mathbf{d}_i = \frac{\mathbf{T}_i}{\mathbf{b}_i} \times \left(\frac{\mathbf{\%Yield} \times \mathbf{\%Efficiency}}{\mathbf{\%Sampling}}\right)_i \times \mathbf{m}_i \qquad \text{.....................(4.3)}$$

<u>Case3</u> If $\mathbf{m}_i \neq 0$ and $\mathbf{x}_i \neq 0$ (operator working with tooling)

$$\mathbf{d}_i = \left(\frac{\mathbf{T}_i}{\mathbf{a}_i + \mathbf{b}_i}\right) \times \left(\frac{\mathbf{\%Yield} \times \mathbf{\%Efficiency}}{\mathbf{\%Sampling}}\right)_i \times \mathbf{m}_i \qquad \text{.................(4.4)}$$

Where:

$$\mathbf{m}_i \leq \left(\frac{\mathbf{a}_i + \mathbf{b}_i}{\mathbf{a}_i}\right) \times \mathbf{x}_i \qquad \text{.....................(4.5)}$$

The explanations of equation 4.4 and 4.5 are presented in the next section.

3. <u>Space</u>

$$\mathbf{s}_i \geq \mathbf{s}_i^{o} \times \mathbf{x}_i \qquad \text{.....................(4.6)}$$

$$\mathbf{s}_i \geq \mathbf{s}_i^{m} \times \mathbf{m}_i \qquad \text{.....................(4.7)}$$

Space factor (s_i) is the factor that the company determines for its space area of each operation by comparing with the station. If one operation can be assigned into one station, its space factor is 1. However, there is the case that some operations need to have space area more than 1 station, so its space factor is indicated as 1.5,2,3, for example.

Note: the space factor is not the integer number.

The equation 4.6 and 4.7 came from the concept that the space factor at each operation is the maximum number between space required for operators and space required for tooling.

$$\sum_{i=1}^{n} s_i \leq \text{Constant} \qquad \text{.....................(4.8)}$$

Total space available in one line must not exceed a constant number, which depends on each company.

4. <u>Bounded on operator</u> (x_i)

$$0 \leq x_i \leq \text{Constant} \qquad \text{.....................(4.9)}$$

Numbers of operator available at each operation must not exceed a constant number, which depends on each operation. It is because some operations need high skill operators that the company owns limit.

$$\sum_{i=1}^{n} x_i \leq \text{Constant} \qquad \text{.....................(4.10)}$$

Total operators available in one line must not exceed a constant number, which depends on the restricted resource of each company.

5. <u>Bounded on tooling</u> (m_i)

$$0 \leq m_i \leq \text{Constant} \qquad \text{.....................(4.11)}$$

Numbers of tooling available at each operation must not exceed a constant number, which depends on the restricted resource of each company.

4.4 Application of the Model to the Study

In this study, the line to assemble HGA can be defined as in **Case 3** of the proposed model where every operator working with the tooling. Looking at these two equations in this case 3 :

$$\mathbf{d_i} = \left(\frac{\mathbf{T_i}}{\mathbf{a_i + b_i}}\right) \times \left(\frac{\mathbf{\%Yield \times \%Efficiency}}{\mathbf{\%Sampling}}\right)_i \times \mathbf{m_i} \qquad \text{.............(4.4)}$$

$$\mathbf{m_i} \leq \left(\frac{\mathbf{a_i + b_i}}{\mathbf{a_i}}\right) \times \mathbf{x_i} \qquad \text{............(4.5)}$$

Equation (4.4)

In equation (4.4) , $\mathbf{d_i}$ is the function of tooling $(\mathbf{m_i})$ from the concept of every output uses the tooling to produce and the numbers of output are counted from the produced output of each tooling, so the numbers of output depend on the numbers of tooling in each operation.

For the total operating time $(\mathbf{T_i})$, it is suitable to determine the total operating time in seconds per day in order to match with the existing data of the company. In this study, the **operating hours** of the company is 21 hours per day. Thus,

$$\mathbf{T_i} = 3600 \text{ (sec/hr.)} \times 21 \text{ (hr./day)} \times \text{ (1 - \% allowance}_i)$$

Where : % allowance$_i$ is the percentage of ability of operator to perform the job in each operation as discussed before. Generally, in this study, this factor is equal to 15%

Therefore, the equation (4.4) is altered to be:

$$\mathbf{d_i} = \left(\frac{75600 \times (1 - \mathbf{\%allowance_i})}{\mathbf{a_i + b_i}}\right) \times \left(\frac{\mathbf{\%Yield \times \%Efficiency}}{\mathbf{\%Sampling}}\right)_i \times \mathbf{m_i}$$

Note : $\mathbf{d_i}$ from this equation implies the numbers of output per day in each operation.

For the cycle time, $\mathbf{a}_i$ is the cycle time of each operator and $\mathbf{b}_i$ is the cycle time of each tooling at operation i. $\mathbf{a}_i$ is not relate to $\mathbf{b}_i$ and both are measured separately. When operator works with tooling, the cycle time must be both the cycle time of operator and cycle time of tooling, i.e. $\mathbf{a}_i + \mathbf{b}_i$.

Equation (4.5)

Equation (4.5) came from the concept of numbers of tooling ($\mathbf{m}_i$) relate to numbers of operator ($\mathbf{x}_i$) in each operation. Since every operator working with tooling, there must be at least the numbers of tooling equal to the numbers of operator, i.e.

$$\mathbf{m}_i = \mathbf{x}_i$$

However, there is the case that one operator can operate more than one tooling. This is because the cycle time of tooling is more that the cycle time of operator and the numbers of added tooling are depended on the fraction between the cycle time of operator and the cycle time of tooling ($\mathbf{a}_i, \mathbf{b}_i$), i.e.

$$\mathbf{m}_i = \mathbf{x}_i + \left(\frac{\mathbf{b}_i}{\mathbf{a}_i}\right) \times \mathbf{x}_i$$

However, the fraction between $\mathbf{b}_i$ and $\mathbf{a}_i$ may not be the integer number, so the $\leq$ is used to round off $\mathbf{m}_i$ to be an integer number, i.e.

$$\mathbf{m}_i \leq \mathbf{x}_i + \left(\frac{\mathbf{b}_i}{\mathbf{a}_i}\right) \times \mathbf{x}_i$$

or

$$\mathbf{m}_i \leq \left(\frac{\mathbf{a}_i + \mathbf{b}_i}{\mathbf{a}_i}\right) \times \mathbf{x}_i \qquad \text{............(4.5)}$$

Since the operator working with the tooling and the numbers of operator and tooling can not be zero ($\mathbf{m}_i$ and $\mathbf{x}_i \neq 0$), there must have been at least one operator and at least one tooling in each operation. However, there is the case that the tooling has no cycle time as we discussed before. This is the case that $\mathbf{b}_i = 0$, so from equation (4.5), it should be that one operator working with one tooling or $\mathbf{m}_i = \mathbf{x}_i$.

However, there are two cases that the tooling has cycle time or $\mathbf{b}_i \neq 0$.

At the operation, if a tooling needs more time to perform its work than an operator needs, or $\mathbf{b}_i > \mathbf{a}_i$. It will be beneficial to allow one operator to operate more than one tooling and then, the fraction of tooling and operator can be computed from equation (4.5).

The other case is that if the tooling has cycle time but this cycle time is less than the cycle time for an operator to deal with his job or $\mathbf{b}_i < \mathbf{a}_i$. From equation (4.5), the fraction value of $\mathbf{a}_i$ and $\mathbf{b}_i$ is less than 1, so after round off, it should be that one operator working with one tooling or $\mathbf{m}_i = \mathbf{x}_i$.

Therefore, it can be conclude that the equation of $\mathbf{m}_i$ and $\mathbf{x}_i$ in equation (4.5) can be used in every case even there is no cycle time of tooling. However, this should be based on the acceptable assumption that each operation has at least one operator and one tooling.

Space factor

In this case study, the space factor is in the form of the station. Therefore, the total space factor must not more than 54 in one line.

Total numbers of operator

In this case study, the company indicates that the total numbers of operator must not be more than 37 in one line.

In conclusion, the most appropriate model for this case study is:

Objective function: Max Final Output (d_n)

S.T.

$$d_{i+1} \leq d_i \qquad\qquad i = 1,2,\ldots\ldots\ldots\ldots,n-1$$

$$d_i = \left(\frac{75600 \times (1 - \%allowance_i)}{a_i + b_i}\right) \times \left(\frac{\%Yield \times \%Efficiency}{\%Sampling}\right)_i \times m_i$$

$$m_i \leq \left(\frac{a_i + b_i}{a_i}\right) \times x_i$$

$$s_i \geq s_i^o \times x_i$$

$$s_i \geq s_i^m \times m_i$$

$$\sum_{i=1}^{n} s_i \leq 54$$

$0 \leq x_i \leq$ number of operators available at operation i

$$\sum_{i=1}^{n} x_i \leq 37$$

$0 \leq m_i \leq$ number of tooling available at each operation