CHAPTER II

THE PROPAGATOR AND FEYNMAN PATH INTEGRAL

Introduction

As discussed in chapter I, the distribution of the density of electron states in the presence of a random potential due to defects is at present a central point in the discussion of the energy spectra of a two-dimensional system in a transverse magnetic field. It is customarily assumed that the density of states of an ideal 2D-electron layer placed in a transverse quantizing magnetic field constitutes a set of equidistant δ functions with spacing equal to the cyclotron energy $\hbar\Omega$. Each Landau level is strongly degenerate, and the multiplicity of this degeneracy (disregarding spin and valley-orbit degeneracy) is equal to $N_L = L^2/2\pi\ell^2$, where $\ell = (\hbar c/eB)^{1/2}$ is the magnetic length. A random potential (of an homogeneity, of an impurity, etc.) lifts the Landau-level degeneracy, this is the cause of their finite width Γ. If this width is related to the short-range potential of the scatterers, the density of states between Landau levels should be vanishingly small. On the basis of recent determinations of the electron states on the Fermi level by measuring the oscillatory dependences of the magnetization, of the electronic specific heat , and also the thermoactivation conductivity, it was concluded that the density of states between Landau levels is exponentially small and is an appreciable fraction of the DOS at B = 0. This conclusion contradicts directly the results of Ando and Uemura. Although there have been several calculations (2,14) of the broadening of the Landau levels due to disorder, none of these theories predict a significant density of states lying between Landau levels. For these reasons, we propose a simple model of disorder to show that the broadening of Landau levels and a significant DOS between Landau levels can be obtained in a simple and a consistent manner. The method used here is very similar to the procedure that Sritrakool et al. (23) have used previously in 3D to explain the origin of Urbach tails in optical absorption near band edges. As before we solve here for n(E) using the path integral method, which at present appears to be the best for obtaining an analytic n(E) for arbitrary L disorder. Before going to the discussion on our work in the next chapter, we devote this chapter to review the Feynman path integral and some solvable problems that can be solved by this theory.

The Propagator and Feynman Path Integral

In quantum mechanics, the dynamical information of a quantum mechanical system is contained in the wave function. It is a function, sometimes called the probability amplitude, that determines the wave associated with a particle. In practice, we can obtain this wave function by solving the Schrodinger' s equation.

In Schrodinger' s picture (24) , there exists the state vector $|\Psi(t)\rangle$ that evolves as

$$|\Psi(t)\rangle = U(t,t')|\Psi(t')\rangle , \qquad (2.1)$$

where $U(t,t')$ is the time evolution operator satisfying the following properties,

i) $$i\hbar \frac{\partial}{\partial t} U(t,t') = HU(t,t')$$

ii) $$U(t,'t') = 1$$

iii) $$U(t'',t)U(t,t') = U(t'',t')$$

iv) $U^{+}(t'',t') = U^{-1}(t'',t') = U(t',t'')$

and H is the Hamiltonian. If the Hamiltonian is not an explicit function of time then the evolution operator is of the form

$$U(t'',t') = \exp[-(i/\hbar)H(t''-t')]. \qquad (2.2)$$

In the configuration representation (2.1) becomes

$$<\vec{x}''|\Psi(t'')> = \int_{-\infty}^{\infty} <\vec{x}''|U(t'',t')|\vec{x}'><\vec{x}'|\Psi(t')>d^3x', \quad (2.3)$$

where the complete set

$$\int_{-\infty}^{\infty} |\vec{x}'><\vec{x}'|\, d^3x' = 1. \qquad (2.4)$$

We can rewrite equation (2.3) as

$$\Psi(\vec{x}',t') = \int_{-\infty}^{\infty} K(\vec{x}'',\vec{x}';t'',t')\Psi(\vec{x}',t')d^3x', \quad (2.5)$$

where $K(\vec{x}'',\vec{x}';t'',t') = <\vec{x}''|U(t'',t')|\vec{x}'>. \qquad (2.6)$

$K(\vec{x}'',\vec{x}';t'',t')$ is called the propagator or the probability amplitude of a particle to go from $\vec{x}'$ at time t' to $\vec{x}''$ at time t''.

According to Feynman ' s ideas (25), there are infinitely many paths for a particle to go from the initial point to the final point under restrictive conditions that

$\vec{x}(t')=\vec{x}'$, $\vec{x}(t'')=\vec{x}''$. Each trajectory contributes to the total amplitude, to go from $\vec{x}'$ to $\vec{x}''$. They contribute equal amounts to the total amplitude, but contribute at different phases. The phase of the contribution from a given path is the action S for that path in units of action $\hbar$. That is, to summarise, the probability $P(\vec{x}'',\vec{x}')$ to go from a point $\vec{x}'$ at t' to the point $\vec{x}''$ at t'' is the absolute square $P(\vec{x}'',\vec{x}') = |K(\vec{x}'',\vec{x}';t'',t')|^2$ of an amplitude $K(\vec{x}'',\vec{x}';t'',t')$ to go from $\vec{x}'$ to $\vec{x}''$. This amplitude is the sum of contributions $\Phi[\vec{x}(t)]$ from each path,

$$K(\vec{x}'',\vec{x}';t'',t') = \sum_{\substack{\text{over all paths} \\ \text{from } \vec{x}' \text{ to } \vec{x}''}} \Phi[\vec{x}(t)] . \tag{2.7}$$

The contribution of a path has a phase proportional to the action S,

$$\Phi[\vec{x}(t)] = [\text{const}]e^{\{(i/\hbar)S[\vec{x}(t)]\}}, \tag{2.8}$$

and

$$S[\vec{x}(t)] = \int_{-\infty}^{\infty} L(\vec{x},\dot{\vec{x}})dt, \tag{2.9}$$

with the Lagrangian

$$L(\vec{x},\dot{\vec{x}}) = (1/2)m\dot{\vec{x}}^2 - V(\vec{x}) . \tag{2.10}$$

Actually, we can not evaluate $K(\vec{x}'',\vec{x}';t'',t')$ from (2.7) directly because of the infinitely many paths contributing. Feynman (25) proposed another way to perform a new formalism of $K(\vec{x}'',\vec{x}';t'',t')$. By dividing the time variable into steps of width $\varepsilon \to 0$, this gives us a set of values t_i spaced at a distance ε apart between the values t' and t''. At each time t_i we select some special $\vec{x}_i$ and construct a path by connecting all points. It is possible to define a sum over all paths in this manner by taking a multiple integral over all values of $\vec{x}_i$ for i between 1 and N-1, where

$$N\varepsilon = t'' - t'$$

$$\varepsilon = t_i - t_{i-1}$$

$$t_0 = t'$$

$$t_N = t''$$

$$\vec{x}_0 = \vec{x}' \; ; \; \vec{x}_N = \vec{x}''.$$

The resulting equation is

$$K(\vec{x}'',\vec{x}';t'',t') = \lim_{N\to\infty} \int\int\ldots\int \frac{e^{(i/\hbar)S[\vec{x}(t)]}}{A} \frac{d^3x_1}{A} \frac{d^3x_2}{A} \ldots \frac{d^3x_{N-1}}{A}, \quad (2.11)$$

where $S[\vec{x}(t)] = \int_{t'}^{t''} L(\vec{x},\dot{\vec{x}})dt$ and the normalizing factor $A = (2\pi i\hbar\varepsilon/m)^{3/2}$.

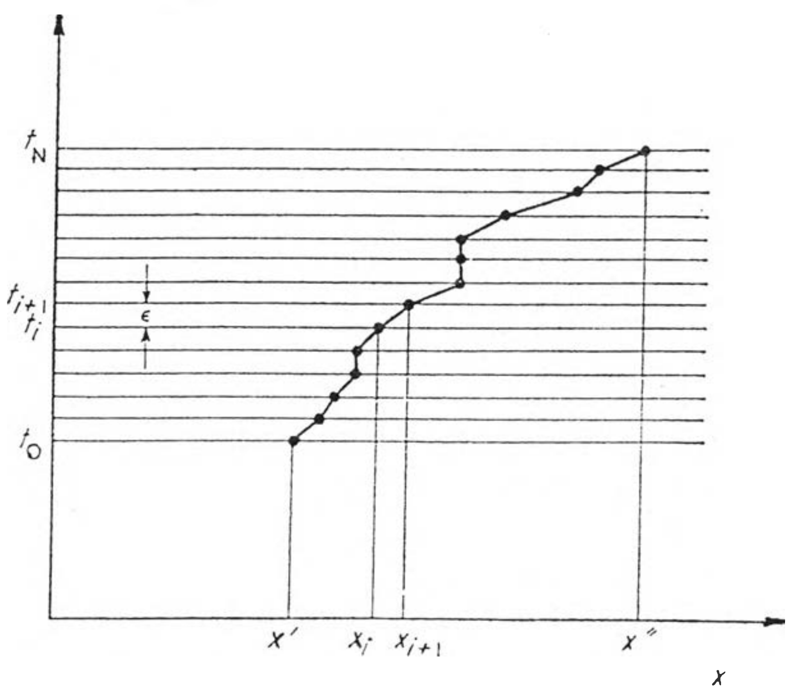

Fig.10 The sum over paths is defined as a limit, in which at first the path is specified by giving only its coordinate x at a large number of specified times separated by very small intervals ε. The path sum is then an intergral over all these specified coordinates. Then to achieve the correct measure, the limit is taken as ε approaches 0.

For small time slices,

$$S(t_i,t_{i-1}) = \int_{t_{i-1}}^{t_i} L(\vec{x},\dot{\vec{x}})\, dt$$

$$\cong (m/2\varepsilon)(\vec{x}_i - \vec{x}_{i-1})^2 - \varepsilon V(\vec{x}_i), \qquad (2.12)$$

so that (2.11) can be written as

$$K(\vec{x}'', \vec{x}'; t'', t') = \lim_{N\to\infty} (m/2\pi i\hbar\varepsilon)^{3N/2}\int\int\ldots\int \exp\,(i/\hbar)[\sum_{i=1}^{N} \{(m/2\varepsilon)(\vec{x}_i - \vec{x}_{i-1})^2$$

$$- \varepsilon V(\vec{x}_i)\}]\, d^3x_1 d^3x_2 \ldots d^3x_{N-1}. \qquad (2.13)$$

Feynman wrote this sum over all paths in a less restrictive notation as

$$K(\vec{x}'', \vec{x}'; t'',t') = \int_{\vec{x}'}^{\vec{x}''} D[\vec{x}(t)]e^{(i/\hbar)S[\vec{x}(t)]}, \qquad (2.14)$$

which is called a path integral.

Path Integral of a Free Particle

From (2.13) we can compute the propagator of a free particle. The Lagrangian for a free particle is

$$L(\vec{x}, \dot{\vec{x}}) = (1/2)m\dot{\vec{x}}^2. \qquad (2.15)$$

The three-dimensional propagator is simply the product of three one-dimensional propagators, so that there is no point in cluttering our equations with vectors. We wish to evaluate

$$K(x'', x'; t'', t') = \lim_{N\to\infty} \iint\ldots\int \exp[(im/2\hbar\varepsilon)\sum_{i=1}^{N} (x_i - x_{i-1})^2](m/2\pi i\hbar\varepsilon)^{N/2}$$

$$dx_1 dx_2 \ldots dx_{N-1} . \qquad (2.16)$$

This is an integral of the form $\int_{-\infty}^{\infty} \exp[-ax^2+bx]\,dx$ which is called a gaussian integral. Since the integral of a gaussian is again gaussian, we may carry out the integrations on one variable after the other with the help of the formula

$$\int_{-\infty}^{\infty} (m/2\pi i\hbar\varepsilon)^{-2/2} \exp[(im/2\hbar\varepsilon)\{(x_2-x_1)^2-(x_1-x_0)^2\}]\,dx_1$$

$$= [m/2\pi i\hbar(2\varepsilon)]^{1/2} \exp[\{m/2i\hbar(2\varepsilon)\}(x_2 - x_0)^2]. \quad (2.17)$$

After the integrations are completed, the limit may be taken. The result is

$$K(x'', x'; t'', t') = [m/(2\pi i\hbar(t''-t'))]^{1/2} \exp[\{im/(2\hbar(t''-t'))\}(x''-x')^2]. \quad (2.18)$$

The Quadratic Lagrangian

In principle, if the path integral is still in a gaussian form, it is possible to carry out the integral over all paths in the way described in the previous section . But in real practice, it is too complicated to perform, for example, the harmonic oscillator problem. We now introduce some additional mathematical techniques which help us to sum over paths in some certain situations. The simplest example to be studied is a quadratic Lagrangian, this corresponds to a case in which the action S involves the path x(t) up to and including the second power.

To illustrate how the method works in a such case, consider a particle whose Lagrangian has the form

$$L(x, \dot{x}, t) = a(t)\dot{x}^2 + b(t)\dot{x}x + c(t)x^2 + d(t)\dot{x} + e(t)x + f(t) . \qquad (2.19)$$

The action is the integral of this function with respect to time between two fixed end points. We wish to determine

$$K(x'', x'; t'', t') = \int \exp[(i/\hbar)\int_{t'}^{t''} L(x, \dot{x}, t)dt]D[x(t)], \qquad (2.20)$$

the integral over all paths which go from (x′,t′) to (x″,t″). Of course, it is possible to carry out this integral over all paths in the way which was first described by dividing the region into short time elements, and so on. But we shall not go through this tedious calculation, since we can determine the most important characteristics of the propagator in the following way.

Let $\bar{x}(t)$ be the classical path between the specified end points. This is the path which is an extremum for the action S. In the notation we have been using

$$S_{cl}[x'', x'] = S[\bar{x}(t)] . \qquad (2.21)$$

We can represent x in terms of $\bar{x}$ and y,

$$x = \bar{x} + y . \qquad (2.22)$$

That is to say, instead of defining a point on the path by its distance x(t) from an arbitrary coordinate axis, we measure instead the deviation y(t) from the classical path, as shown in Fig. 11.

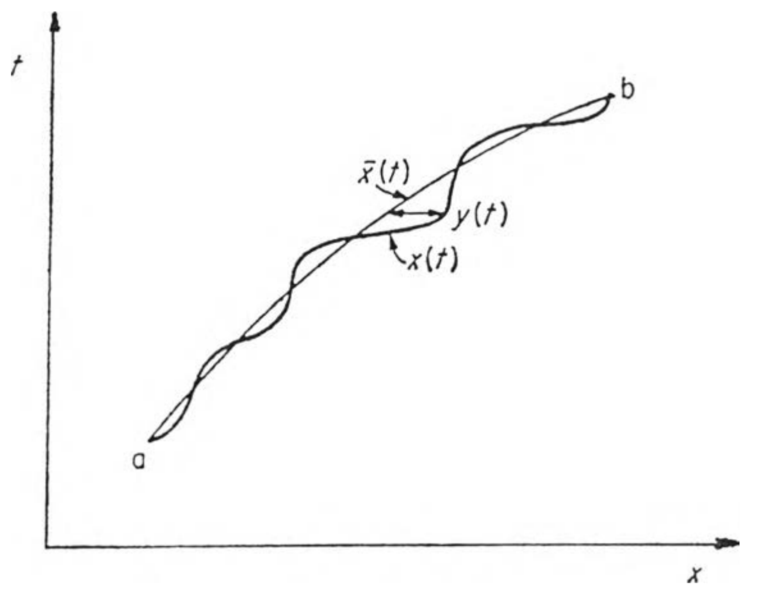

Fig.11 The difference between the classical path $\bar{x}(t)$ and some possible alternative path y(t). The end points $y(t'') = y(t') = 0$.

At each t the variable x and y differ by the constant $\bar{x}$. Therefore, clearly, $dx_i = dy_i$ for each specific point t_i in the subdivision of time. In general, we may say $D[x(t)] = D[y(t)]$. The integral for the action can be written as

$$S[x(t)] \quad = \quad S[\bar{x}(t)+y(t)] = \int_{t'}^{t''}[a(t)(\dot{\bar{x}}^2 + 2\dot{\bar{x}}\dot{y} + \dot{y}^2) + \ldots]dt \,. \quad (2.23)$$

If all terms which do not involve y are collected, the result is just $S[\bar{x}(t)] = S_{cl}[\bar{x}(t)]$. If all terms which contain y as linear factors are collected, the resulting integral vanishes. This could be proved by actually carrying the integration, however, such a calculation is unnecessary, since we already know the result is true. The function $\bar{x}(t)$ is determined by this is very requirement. That is, $\bar{x}$ is so chosen that there is no change in S, to first order, for variations of the path around $\bar{x}$. All that remains are the second order terms in y. These can be easily picked out, so that we can write

$$S[x(t)] \quad = \quad S_{cl}[x'',x'] + \int_{t'}^{t''} [\, a(t)\dot{y}^2 + b(t)y\dot{y} + c(t)y^2 \,]dt. \quad (2.24)$$

The integral over paths does not depend upon the classical path, so that the propagator can be written as

$$K(x'',x';t'',t') \;=\; \exp\{(i/\hbar)S_{cl}[x'',x']\}\int_0^0 \exp[(i/\hbar)\int_{t'}^{t''}(a(t)\dot{y}^2+b(t)y\dot{y}+c(t)y^2)]D[y(t)] \quad (2.25)$$

Since all paths y(t) start from and return to the point y = 0, the integral over paths can be a function only of time at the end points. This means that the propagator can be written as

$$K(x'',x';t'',t') \quad = \quad F(t'',t')\exp[(i/\hbar)S_{cl}[x'',x']]. \quad (2.26)$$

So K is determined except for a multiplying factor F(t″,t′) which may be determined by some other known property of the solution. However, for a quadratic

Lagrangian, van Vleck (26) and Pauli (27) had verified that the pre-factor F(t″, t′) can be evaluated exactly by using the formula

$$F(t'', t') \quad = \quad \det\left[(i/2\pi\hbar)\frac{\partial^2}{\partial x''\partial x'}(S_{cl}[x'', x'])\right]^{1/2}, \qquad (2.27)$$

so that (2.26) becomes

$$K(x'',x';t'',t') \quad = \quad \det\left[(i/2\pi\hbar)\frac{\partial^2}{\partial x''\partial x'}(S_{cl}[x'',x'])\right]^{1/2}\exp[(i/\hbar)S_{cl}[x'',x']]. \quad (2.28)$$

It is interesting to note that the expression $K \sim \exp[(i/\hbar)S_{cl}]$ is exact for the case that S is a quadratic form.

Exact Propagator of a Two-Dimensional Random System

We now consider the problem of an electron confined in two dimensions under the influence of a homogeneous transverse magnetic field, a time varying electric field and a random potential. However, in this problem we represent the random potential by a nonlocal harmonic oscillator which was first introduced by Feynman (28) in his evaluation of the polaron problem. The model we use here is rather an idealized one, but it may be related to the behaviour of an electron moving in an external magnetic field, electric field and disorder potential which is of present interest. Although the problem is one of mathematical complexity, the Lagrangian of this system is quadratic, and the propagator can be evaluated analytically.

The Lagrangian of the electron subject to a combination of all potentials mentioned above with the magnetic field taken in z-direction, is given by

$$L(\vec{r},\dot{\vec{r}},t) = \frac{m}{2}[\dot{x}^2+\dot{y}^2+\Omega(x\dot{y}-y\dot{x})] - \frac{m}{2T}\frac{\omega^2}{2} \times \int_0^T [\vec{r}(t)-\vec{r}(\sigma)]^2 d\sigma + e\vec{E}(t).\vec{r}(t) \tag{2.29}$$

Where $\vec{r} = (x, y)$, $\Omega = \frac{eB}{mc}$ is the cyclotron frequency, ω denotes the nonlocal oscillator frequency and $\vec{E}(t)$ represents the time varying electric field. The required propagator can be written in the path integral form,

$$K(\vec{r}_b, \vec{r}_a; T) = \int_a^b e^{\frac{i}{\hbar}S[b,a]} D[\vec{r}(t)] \tag{2.30}$$

where $S[b, a] = \int_o^T L(\vec{r}, \dot{\vec{r}}, t)\, dt$ is the action, $D[\vec{r}(t)]$ denotes the measure of the path integral to be carried out with the boundary conditions $\vec{r}(o) = \vec{r}_a$ and $\vec{r}(T) = \vec{r}_b$. The propagator in equation (2.30) can be rewritten as

$$K(\vec{r}_b,\vec{r}_a; T) = \int\int D[x(t)]D[y(t)]\exp[\frac{i}{\hbar}(\frac{m}{2}\int_0^T(\dot{x}^2+\dot{y}^2+\Omega(x\dot{y}-y\dot{x}) - \omega^2(x^2+y^2))\, dt$$

$$+\frac{m}{2T}\omega^2\left[\int_0^T\int_0^T x(t)\,x(\sigma)\,dt\,d\sigma+\int_0^T\int_0^T y(t)\,y(\sigma)\,dt\,d\sigma\right]$$
$$+e\left[\int_0^T E_x(t)\,x(t)\,dt+\int_0^T E_y(t)\,y(t)\,dt\right])\Big] \qquad (2.31)$$

$E_x(t)$ and $E_y(t)$ denote the components of $\vec{E}(t)$ in the x and y directions respectively. Unfortunately, the Lagrangian of such a system is rather complicated and the path integration can not be carried out directly. To avoid this difficulty we follow Stratonovich (29) by using the identity

$$\exp\left(\frac{i}{\hbar}\frac{m}{2T}\omega^2\left(\int_0^T\int_0^T x(t)\,x(\sigma)\,dt\,d\sigma+\int_0^T\int_0^T y(t)\,y(\sigma)\,dt\,d\sigma\right)\right)$$
$$=\left\langle \exp\left(\frac{i}{\hbar}\left(f_x\int_0^T x(t)\,dt+f_y\int_0^T y(t)\,dt\right)\right)\right\rangle_{f_x,\,f_y} \qquad (2.32)$$

to transform equation (2.31) into a soluble form. Applying (2.32) to (2.31) we have

$$K(\vec{r}_b,\vec{r}_a;T) = \left\langle K_{eff}(\vec{r}_b,\vec{r}_a;T)\right\rangle_{f_x,\,f_y} \qquad (2.33)$$

where $K_{eff}(\vec{r}_b,\vec{r}_a;T)$ is the effective propagator. This propagator corresponds to the system of an electron moving in the presence of a magnetic field and forced harmonic oscillator potential and is given by

$$K_{eff}(\vec{r}_b,\vec{r}_a;T) = \int\int D[x(t)]\,D[y(t)]\exp\Big[\frac{i}{\hbar}\Big(\frac{m}{2}\int_0^T(\dot{x}^2+\dot{y}^2+\Omega(x\dot{y}-y\dot{x})$$

$$- \omega^2(x^2 + y^2)) dt + \int_0^T (eE_x(t) + f_x) x (t) dt + \int_0^T (eE_y (t) + f_y) y (t) dt.)]$$

(2.34)

The notation <..........> $_{f_x, f_y}$ denotes the Gaussian average defined by

$$< A >_{f_x, f_y} = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp [- \frac{iT}{2\hbar m\omega^2} (f_x^2 + f_y^2)] A df_x df_y}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp [- \frac{iT}{2m\hbar\omega^2} (f_x^2 + f_y^2)] df_x df_y} \quad (2.35)$$

Using the van Vleck - Pauli result, we can write the effective propagator as

$$K_{eff} (\vec{r}_b, \vec{r}_a; T) = F_{eff} (T, 0) \exp [\frac{i}{\hbar} S_{eff}^{cl} (\vec{r}_b, \vec{r}_a; T)] \quad (2.36)$$

where $F_{eff}(T,O)$ is the pre-exponential factor associated with the effective classical action

$$S_{eff}^{cl} (\vec{r}_b, \vec{r}_a; T) = \int_0^T L_{eff} (\vec{r}_{cl}, \dot{\vec{r}}_{cl}, t) dt \quad (2.37)$$

and is evaluated by using the formula

$$F_{eff}(T, O) = \det\left[\frac{i}{2\pi\hbar}\frac{\partial^2}{\partial\vec{r}_a\partial\vec{r}_b}(S^{cl}_{eff}(\vec{r}_b,\vec{r}_a;T))\right]^{\frac{1}{2}}. \tag{2.38}$$

The $\vec{r}_{cl}$ appearing in equation (2.37) represents the classical path of an electron in the effective system which corresponds to an effective Lagrangian,

$$L_{eff}(\vec{r},\dot{\vec{r}},t) = \frac{m}{2}[\dot{x}^2+\dot{y}^2+\Omega(x\dot{y}-y\dot{x}) - \omega^2(x^2+y^2)]$$

$$+ (e\vec{E}_x(t) + f_x)x + (eE_y(t) + f_y)y. \tag{2.39}$$

A. Effective classical action

We now wish to calculate the effective classical action $S^{cl}_{eff}(\vec{r}_b,\vec{r}_a;T)$

corresponding to an effective Largrangian $L_{eff}(\vec{r},\dot{\vec{r}}\ t)$. To simplify this problem we employ the 2 x 2 matrices introduced by Papadoupoulos (30) in his work on the magnetization of harmonically bound charges

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.40}$$

which obey the relation $J^2 = -I$. Let us represent the component of $\vec{r}$ perpendicular to the magnetic field by a 2 x 1 matrix

$$r_\perp = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{2.41}$$

Similarly, the driving force of the effective system can be replaced by $F_\perp$,

$$F_\perp = f_\perp + eE_\perp = \begin{pmatrix} f_x + eE_x(t) \\ f_y + eE_y(t) \end{pmatrix} \tag{2.42}$$

From (2.39), (2.40), (2.41) and (2.42), we see that the effective Lagrangian in matrix form can be written as

$$L_{eff}(\vec{r}, \dot{\vec{r}}, t) = \frac{m}{2}(\dot{r}_\perp^2 - \omega^2 r_\perp^2 - \Omega \tilde{r}_\perp J \dot{r}_\perp) + \tilde{F}_\perp r_\perp \tag{2.43}$$

where $\tilde{r}_\perp$ and $\tilde{F}_\perp$ denote the transpose of $r_\perp$ and $F_\perp$ respectively. Equation (2.43) leads to the equation of classical motion

$$\ddot{r}_\perp + \Omega J \dot{r}_\perp + \omega^2 I r_\perp = \frac{F_\perp}{m} \tag{2.44}$$

subject to the boundary conditions

$$r_\perp(0) = r'_\perp = \begin{pmatrix} x_a \\ y_a \end{pmatrix} \qquad r_\perp(T) = r''_\perp = \begin{pmatrix} x_b \\ y_b \end{pmatrix} \tag{2.45}$$

The solution of (2.44) can be split into two parts,

$$r_\perp(t) = r_{\perp C}(t) + r_{\perp P}(t) \tag{2.46}$$

where $r_{\perp C}(t)$ is the solution of a homogenous equation,

$$\ddot{r}_\perp + \Omega J \dot{r}_\perp + \omega^2 I r_\perp = 0 \tag{2.47}$$

The auxiliary equation associated with (2.47) is

$$R^2 + \Omega J R + \omega^2 I = 0 \tag{2.48}$$

which is satisfied by the matrices

$$R_1 = -\frac{\Omega}{2} J + \Omega' J$$

$$R_2 = -\frac{\Omega}{2} J - \Omega' J \tag{2.49}$$

where $\Omega' = \left(\frac{\Omega^2}{4} + \omega^2\right)^{\frac{1}{2}}$. We then have

$$r_{\perp C}(t) = e^{-\frac{\Omega}{2} J t} \left(e^{\Omega' J t} A + e^{-\Omega' J t} B \right) , \tag{2.50}$$

where A and B are arbitrary constants. Note that a matrix of the form $e^{\pm J \varphi}$ has the property

$$e^{\pm J \varphi} = I \cos \varphi \pm J \sin \varphi \tag{2.51}$$

After applying the boundary conditions, equation (2.50) becomes

$$r_{\perp C}(t) = \frac{e^{-\frac{\Omega}{2} J t}}{\sin(\Omega' T)} \left[e^{\frac{\Omega}{2} J T} \sin(\Omega' t) r''_{\perp} + \sin(\Omega'(T - t)) r'_{\perp} \right] . \tag{2.52}$$

We now consider $r_{\perp P}(t)$ which satisfies the inhomogenous equation

$$(D^2 + \Omega J D + \Omega^2 I) r_\perp = \frac{F_\perp}{m} \qquad (2.53)$$

where D denotes the differential operator $\frac{d}{dt}$. To calculate $r_{\perp P}(t)$ we shall use the Green ' s function method. After getting the Green ' s function $g(t, s)$ which obeys the relation

$$(D^2 + \Omega J D + \omega^2 I) g (t, s) = \delta (t - s) \qquad (2.54)$$

we obtain the solution of $r_{\perp P}(t)$. The result is

$$r_{\perp P}(t) = \int_0^T g (t, s) \frac{F_\perp(s)}{m} ds . \qquad (2.55)$$

The Green ' s function $g(t, s)$ can be evaluated exactly from (2.54) .

We find

$$g (t, s) = - \frac{1}{\Omega' \sin (\Omega' T)} [H (t - s) \sin (\Omega' s) \sin (\Omega'(T - t))$$

$$+ H (s- t) \sin (\Omega' t) \sin (\Omega' (T - s))] e^{\frac{\Omega}{2} J (s - t)} \qquad (2.56)$$

with H(t-s) and H(s-t) being the Heaviside step function obeying the relation

$$H (x) = 1, x > 0$$

$$= 0, x < 0 \qquad (2.57)$$

Finally we obtain the general solution

$$r_\perp(t) = r_{\perp C}(t) + r_{\perp P}(t)$$

$$= \frac{e^{-\frac{\Omega}{2}Jt}}{\sin\Omega' T}\left(e^{\frac{\Omega}{2}JT}\sin(\Omega' t)\, r''_\perp + \sin(\Omega'(T-t))\, r'_\perp\right)$$

$$+ \int_0^T g(t,s)\frac{F_\perp(s)}{m}\, ds. \tag{2.58}$$

We now focus our attention on the effective classical action

$$S^{cl}_{eff}(\vec{r}_b, \vec{r}_a; T),$$

$$S^{cl}_{eff}(\vec{r}_b, \vec{r}_a; T) = \frac{m}{2}\int_0^T \dot{r}_\perp^2\, dt - \frac{m}{2}\int_0^T (\Omega \tilde{r}_\perp J \dot{r}_\perp + \omega^2 r_\perp^2)\, dt$$

$$+ \int_0^T \tilde{F}_\perp r_\perp\, dt. \tag{2.59}$$

Integrating by parts the first term of (2.59) and applying the equation of motion (2.44), we obtain

$$S^{cl}_{eff}(\vec{r}_b, \vec{r}_a; T) = \frac{m}{2}(\tilde{r}_\perp(T)\dot{r}_\perp(T) - \tilde{r}_\perp(0)\dot{r}_\perp(0)) + \frac{1}{2}\int_0^T \tilde{r}_\perp F_\perp\, dt\,. \tag{2.60}$$

The complete solution for the effective action is

$$S^{cl}_{eff}(\vec{r}_b, \vec{r}_a; T) = \frac{m}{2} \frac{\Omega'}{\sin(\Omega' t)} \left(\cos(\Omega' T)(r''^{2}_{\perp} + r'^{2}_{\perp}) - 2\tilde{r}''_{\perp} e^{\frac{-\Omega}{2} J T} r'_{\perp} \right) + \int_0^T \tilde{r}_{\perp C}(t) F_{\perp} dt + \frac{1}{2} \int_0^T \int_0^T \tilde{F}_{\perp}(t) g(t, s) \frac{F_{\perp}(s)}{m} ds dt . \tag{2.61}$$

The pre-exponential factor associated with the effective propagator can be evaluated exactly. It is found that

$$F_{eff}(T, 0) = \frac{m \Omega'}{2\pi i \hbar \sin(\Omega' T)} . \tag{2.62}$$

From (2.36), (2.51) and (2.62) we obtain the effective propagator

$$K_{eff}(\vec{r}_b, \vec{r}_a; T) = \left(\frac{m \Omega'}{2\pi i \hbar \sin(\Omega' T)} \right) \exp\left[\frac{i}{\hbar} \left(\frac{m \Omega'}{2 \sin(\Omega' T)} \left[\cos(\Omega' T) \times (r''^{2}_{\perp} + r'^{2}_{\perp}) - 2\tilde{r}''_{\perp} e^{\frac{-\Omega}{2} J T} r'_{\perp} \right] + \int_0^T \tilde{r}_{\perp C}(t) F_{\perp}(t) dt + \frac{1}{2m} \int_0^T \int_0^T \tilde{F}_{\perp}(t) g(t, s) F_{\perp}(s) ds dt \right) \right] \tag{2.63}$$

B. Exact propagator

Let us now go back to the original propagator. From (2.33) we have

$$K(\vec{r}_b,\vec{r}_a;T) = \langle K_{eff}(\vec{r}_b,\vec{r}_a;T)\rangle_{f_x,\, f_y} \qquad (2.64)$$

By splitting $F_\perp$ into two parts, $F_\perp = f_\perp + e\,E_\perp$ and then performing the Gaussian average we obtain the desired propagator

$$K(\vec{r}_b,\vec{r}_a;T) = F(T)\exp\Big[\frac{i}{\hbar}\Big(S_0^{cl}(\vec{r}_b,\vec{r}_a;T) + e\int_0^T \tilde{R}_{\perp C}(t)\,E_\perp(t)\,dt + \frac{e^2}{2m}\int_0^T\!\!\int_0^T \tilde{E}_\perp(t)\,G(t,s)\,E_\perp(s)\,ds\,dt\Big)\Big] \qquad (2.65)$$

where

$$S_o^{cl}(\vec{r}_b,\vec{r}_a;T) = \frac{m\Omega'}{2\sin(\Omega' t)}\Big[\cos(\Omega' T)\,(r_\perp^{2\,\prime\prime} + r_\perp^{2\,\prime}) - 2\,\tilde{r}_\perp^{\,\prime\prime}\, e^{\frac{-\Omega J T}{2}}\, r_\perp^{\prime}\Big]$$

$$+ \frac{m\omega^4 \sin(\Omega' T)}{4\Omega'\big(\cos(\frac{\Omega}{2}T) - \cos(\Omega' T)\big)}\int_0^T\!\!\int_0^T \tilde{r}_{\perp C}(t)\, r_{\perp C}(s)\, ds\,dt$$

$$= \frac{m\Omega'}{2\sin(\Omega' T)}\Big[\cos(\Omega' T)\,(r_\perp^{2\,\prime\prime} + r_\perp^{2\,\prime}) - 2\,\tilde{r}_\perp^{\,\prime\prime}\, e^{\frac{-\Omega}{2} J T}\, r_\perp^{\prime}\Big]$$

$$+ \frac{m\Omega'}{4\sin(\Omega' T)}\Big[\big(\cos(\frac{\Omega}{2}T) - \cos(\Omega' T)\big)\,(r_\perp^{\prime\prime} + r_\perp^{\prime})^2$$

$$+ \frac{4}{\Omega'} \left(\Omega' \sin\left(\frac{\Omega T}{2}\right) - \frac{\Omega}{2} \sin(\Omega' T) \right) \tilde{r}'_{\perp} J r''_{\perp}$$

$$+ \frac{1}{\Omega'^2} \frac{\left(\Omega' \sin\left(\frac{\Omega T}{2}\right) - \frac{\Omega}{2} \sin(\Omega' T) \right)^2}{\left(\cos\left(\frac{\Omega T}{2}\right) - \cos(\Omega' T) \right)} (r''_{\perp} - r'_{\perp})^2]$$

(2.66)

$$\tilde{R}_{\perp C}(t) = \tilde{r}_{\perp C}(t) + \frac{\omega^4 \sin(\Omega' T)}{2\Omega' \left(\cos\left(\frac{\Omega T}{2}\right) - \cos(\Omega' T) \right)} \int_0^T \tilde{r}_{\perp C}(\sigma)\, d\sigma \int_0^T g(s, t)\, ds$$

$$= \frac{1}{\sin(\Omega' T)} \left(\sin(\Omega'(T - t))\, \tilde{r}'_{\perp} + \sin(\Omega' t)\, \tilde{r}''_{\perp} e^{\frac{-\Omega}{2} J T} \right) e^{\frac{\Omega}{2} J t}$$

$$- \frac{1}{2 \sin(\Omega' T)} \left[(\tilde{r}''_{\perp} + \tilde{r}'_{\perp}) - \frac{1}{\Omega'} \frac{\left(\Omega' \sin\left(\frac{\Omega T}{2}\right) - \frac{\Omega}{2} \sin(\Omega' T) \right)}{\left(\cos\left(\frac{\Omega T}{2}\right) - \cos(\Omega' T) \right)} \right.$$

$$\times (\tilde{r}''_{\perp} - \tilde{r}'_{\perp}) J \Big] \left[\left(\sin(\Omega'(T - t)) + \sin(\Omega' t)\, e^{\frac{-\Omega}{2} J T} \right) e^{\frac{\Omega}{2} J t} \right.$$

$$\left. - \sin(\Omega' T) \right]$$

(2.67)

$$G(t,s) = g(t,s) + \frac{\omega^4 \sin(\Omega' T)}{2\Omega' \left(\cos\left(\frac{\Omega T}{2}\right) - \cos(\Omega' T) \right)} \int_0^T \int_0^T g(t,t')\, g(s',s)\, dt'\, ds'$$

$$= -\frac{1}{\Omega' \sin(\Omega' T)} [\, H(t-s) \sin(\Omega' s) \sin(\Omega'(T-t))$$

$$+ H(s-t) \sin(\Omega' t) \sin(\Omega'(T-s)) \,]\, e^{\frac{\Omega}{2} J(s-t)}$$

$$+ \left[\frac{1}{2\Omega' \sin(\Omega' T) \left(\cos\left(\frac{\Omega T}{2}\right) - \cos(\Omega' T) \right)} \right] [\, e^{\frac{-\Omega}{2} Jt}$$

$$\times \ (\sin \Omega' (T-t))$$

$$+ \sin(\Omega' t)\, e^{\frac{\Omega}{2} JT} - \sin(\Omega' T) \,]\, [\, e^{\frac{\Omega}{2} Js} \ (\sin(\Omega'(T-s))$$

$$+ \sin(\Omega' s)\, e^{\frac{-\Omega}{2} JT}) - \sin(\Omega' T) \,]$$

(2.68)

with $F(T)$

$$= \frac{m T \omega^2}{4 i \hbar \pi \left(\cos\left(\frac{\Omega T}{2}\right) - \cos(\Omega' T) \right)}$$

$$= \frac{m T \omega^2}{\pi\, 8 i \hbar \sin\left(\frac{1}{2}\left(\Omega' + \frac{\Omega}{2}\right) T\right) \sin\left(\frac{1}{2}\left(\Omega' - \frac{\Omega}{2}\right) T\right)}$$

(2.69)

We now consider the special case when the external electric field $\vec{E}(t)$ is zero. Equation (2.65) reduces to

$$K(\vec{r}_b,\vec{r}_a;T) = \frac{mT\omega^2}{4i\hbar\pi\left(\cos\left(\frac{\Omega T}{2}\right) - \cos(\Omega' T)\right)} \exp\Big[\frac{i}{\hbar}\Big(\frac{m\Omega'}{2\sin(\Omega' T)}$$

$$\times\left[\cos(\Omega' T)\left(r_\perp^{''2} + r_\perp^{'2}\right) - 2\tilde{r}_\perp^{''} e^{\frac{-\Omega}{2}JT} r_\perp^{'}\right]$$

$$+\frac{m\Omega'}{4\sin(\Omega' T)}\Big[\left(\cos\left(\frac{\Omega T}{2}\right) - \cos(\Omega' T)\right)\left(r_\perp^{'} + r_\perp^{''}\right)^2$$

$$+\frac{4}{\Omega'}\left(\Omega'\sin\left(\frac{\Omega T}{2}\right) - \frac{\Omega}{2}\sin(\Omega' T)\right)\tilde{r}_\perp^{'} J r_\perp^{''}$$

$$+\frac{1}{\Omega'^2}\frac{\left(\Omega'\sin\left(\frac{\Omega T}{2}\right) - \frac{\Omega}{2}\sin(\Omega' T)\right)^2\left(r_\perp^{''} - r_\perp^{'}\right)^2}{\left(\cos\left(\frac{\Omega T}{2}\right) - \cos(\Omega' T)\right)}\Big]\Big)\Big] \tag{2.70}$$

By using the identities $e^{\pm J\varphi} = I\cos\varphi \pm J\sin\varphi$, $r_\perp^{'} = \begin{pmatrix} x_a \\ y_a \end{pmatrix}$

and $r_\perp^{''} = \begin{pmatrix} x_b \\ y_b \end{pmatrix}$ equation (2.70) can be written in a usual representation as

$$K(\vec{r}_b,\vec{r}_a;T) = \frac{mT\omega^2}{4\pi i\hbar\left(\cos\left(\frac{\Omega}{2}T\right) - \cos(\Omega' T)\right)} \exp\Big[\frac{i}{\hbar}\Big(\frac{m\Omega'}{2\sin(\Omega' T)}$$

$$\times\left[\left(x_b^2 + y_b^2 + x_a^2 + y_a^2\right)\cos(\Omega' T) + 2\sin\left(\frac{\Omega T}{2}\right)\left(x_a y_b - x_b y_a\right)\right.$$

$$- \cos\left(\frac{\Omega}{2}T\right)(x_a x_b + y_a y_b))\,]$$

$$+ \frac{m\,\Omega'}{4\sin(\Omega' T)}\Big[\Big(\cos\left(\frac{\Omega T}{2}\right) - \cos(\Omega' T)\Big)\big((x_b + x_a)^2 + (y_b + y_a)^2\big)$$

$$+ \frac{4}{\Omega'}\Big(\Omega' \sin\left(\frac{\Omega T}{2}\right) - \frac{\Omega}{2}\sin(\Omega' T)\Big)(x_b y_a - x_a y_b)$$

$$+ \frac{1}{\Omega'^2}\,\frac{\Big(\Omega' \sin\left(\frac{\Omega T}{2}\right) - \frac{\Omega}{2}\sin(\Omega' T)\Big)^2}{\Big(\cos\left(\frac{\Omega T}{2}\right) - \cos(\Omega' T)\Big)}\big((x_b - x_a)^2 + (y_b - y_a)^2\big)\,]\,)\,]$$

(2.71)

This is the exact propagator of an electron moving in two dimensions under the influence of a transverse magnetic field and a nonlocal harmonic oscillator potential. We consider the two limiting cases :

a) When the nonlocal harmonic oscillator potential approaches zero, the system of interest corresponds to the case when $\omega \rightarrow 0$, and $\Omega' \rightarrow \frac{\Omega}{2}$. First we consider the pre-exponential factor of (2.71)

$$F(T) = \frac{m\,T\,\omega^2}{4\pi i\hbar\Big(\cos\left(\frac{\Omega T}{2}\right) - \cos(\Omega' T)\Big)} \qquad (2.72)$$

If we take the limit $\Omega' \rightarrow \frac{\Omega}{2}$ or $\omega \rightarrow 0$, then

$$\lim_{\substack{\omega \to 0 \\ \Omega' \to \frac{\Omega}{2}}} F(T) = \lim_{\Omega' \to \frac{\Omega}{2}} \frac{m T (\Omega' + \frac{\Omega}{2})(\Omega' - \frac{\Omega}{2})}{8 \pi i \hbar \sin(\frac{1}{2}(\Omega' - \frac{\Omega}{2}) T) \sin(\frac{1}{2}(\Omega' + \frac{\Omega}{2}) T)}$$

$$= \left(\frac{m}{2 \pi i \hbar T}\right)\left(\frac{\Omega T}{2 \sin(\frac{\Omega T}{2})}\right) \qquad (2.73)$$

(Note that $\omega^2 = \Omega'^2 - \frac{\Omega^2}{4} = (\Omega' - \frac{\Omega}{2})(\Omega' + \frac{\Omega}{2})$.) One can easily verify that when

$\Omega' \to \frac{\Omega}{2}$ the exponential term of (2.71) reduces to

$$\exp\left[\frac{i}{\hbar} \frac{m\Omega}{4 \sin(\frac{\Omega T}{2})} \left(\cos(\frac{\Omega T}{2})(x_b^2 + x_a^2 + y_b^2 + y_a^2) + 2\left[\sin(\frac{\Omega T}{2})\right.\right.\right.$$

$$\left.\left.\left. \times (x_a y_b - x_b y_a) - \cos(\frac{\Omega T}{2})(x_b x_a + y_b y_a)\right]\right)\right]$$

$$= \exp\left[\frac{i}{\hbar}\frac{m}{2}\left(\frac{\Omega}{2}\cot(\frac{\Omega T}{2})\left((x_b - x_a)^2 + (y_b - y_a)^2 + \Omega(x_a y_b - x_b y_a)\right)\right)\right]$$

$$(2.74)$$

Thus, from (2.73) and (2.74) we get, when $\omega \to 0$

$$K(\vec{r}_b, \vec{r}_a; T) = \left(\frac{m}{2 \pi i \hbar T}\right)\left(\frac{\Omega T}{2 \sin(\frac{\Omega T}{2})}\right) \exp\left[\frac{mi}{2\hbar}\left(\frac{\Omega}{2}\cot(\frac{\Omega T}{2})\left[(x_b - x_a)^2\right.\right.\right.$$

$$\left.\left.\left. + (y_b - y_a)^2\right] + \Omega(x_a y_b - x_b y_a)\right)\right] \qquad (2.75)$$

This is the propagator of a charged particle confined in two dimensions in the presence of a transverse magnetic field.

b) When the magnetic field goes to zero, this limiting case corresponds to the case when $\Omega \to 0$ and $\Omega' \to \omega$. We first consider the pre-exponential factor of (2.71) . In this case

$$\lim_{\substack{\Omega\to 0\\ \Omega'\to\omega}} F(T) = \lim_{\substack{\Omega\to 0\\ \Omega'\to\omega}} \frac{mT\omega^2}{8\pi i\hbar \sin(\frac{1}{2}(\Omega' - \frac{\Omega}{2})T)\sin(\frac{1}{2}(\Omega' + \frac{\Omega}{2})T)}$$

$$= \left(\frac{m}{2\pi i\hbar T}\right)\left(\frac{\omega T}{2\sin(\frac{\omega T}{2})}\right)^2$$

(2.76)

and the exponential term of (2.71) when $\Omega \to 0$ becomes

$$\exp\Big[\frac{i}{\hbar}\Big(\frac{m\omega}{2\sin(\omega T)}[(x_b^2 + x_a^2 + y_a^2 + y_b^2)\cos(\omega T) - 2(x_b x_a + y_b y_a)]$$

$$+ \frac{m\omega}{4\sin(\omega T)}[(1-\cos(\omega T))((x_b + x_a)^2 + (y_b + y_a)^2)]\Big)\Big]$$

$$= \exp\left(\frac{i}{\hbar}\frac{m\omega}{4}\cot\left(\frac{\omega T}{2}\right)[(x_b - x_a)^2 + (y_b - y_a)^2]\right).$$

(2.77)

From (2.76) and (2.77), it follows that

$$K(\vec{r}_b, \vec{r}_a; T) = \left(\frac{m}{2\pi i \hbar T}\right)\left(\frac{\omega T}{2\sin\left(\frac{\omega T}{2}\right)}\right)^2 \exp\left(\frac{i}{\hbar}\frac{m\omega}{4}\cot\left(\frac{\omega T}{2}\right)\right.$$

$$\left. \times \left[(x_b - x_a)^2 + (y_b - y_a)^2\right]\right)$$

(2.78)

This is the propagator of a particle confined in two dimensions under the influence of a nonlocal harmonic oscillator potential (31).

The Density of States

If we have a function N(E) which is the total number of states at a given energy E of an electron-atom system, then a function which is called the density of states is defined by

$$n(E) = \frac{d\,N(E)}{dE}, \qquad (2.79)$$

or, equivalently,

$$n(E) = (1/V)\sum_{n=1}^{\infty} \delta(E - E_n), \qquad (2.80)$$

where E_n is the energy of the nth eigenstate, V is the volume of the system. If the system is disordered, we must average (2.80) over the statistical ensemble for the random potential. It is convenient to consider the density of states in the form of (2.80), and in order to apply the path integral method to (2.80), one converts the right hand side of (2.80) into an integral form to get (31),

$$n(E) = (1/2\pi\hbar) \int_{-\infty}^{\infty} e^{[(i/\hbar)ET]} \mathrm{Tr}\, K(\vec{x}'',\vec{x}'; T)dT, \qquad (2.81)$$

where the operator Tr denotes the trace of K. The function K is a retarded propagator describing the propagation of an electron from point $\vec{x}'$ to point $\vec{x}''$, and where $\vec{x}'$ and $\vec{x}''$ are vector positions of the electron in d dimensions. If the propagator K is invariant under translation of $\vec{x}$, this means that

$$K(\vec{x}'',\vec{x}';T) = K(\vec{x}''-\vec{x}';T), \qquad (2.82)$$

so that for finding the density of states, the end point $\vec{x}''$ and the initial point $\vec{x}'$ must be the same. It therefore follows that

$$n(E) = (V/2\pi\hbar) \int_{-\infty}^{\infty} e^{[(i/\hbar)ET]} K(0,0;T)dT. \qquad (2.83)$$

Now applying (2.83) to (2.71) , one finds the density of states of an electron in the absence of an electric field,

$$n(E) = \frac{\alpha\beta m A}{2\pi} \sum_{s=0}^{\infty} \sum_{n=0}^{\infty} \delta'[E-(s+\frac{1}{2})\hbar\alpha-(n+\frac{1}{2})\hbar\beta],$$

(2.84)

where δ' denotes the derivative of the Dirac delta function with respect to its argument, A is the area of the sample $\alpha = \Omega' + \frac{\Omega}{2}$ and $\beta = \Omega' - \frac{\Omega}{2}$. After employing the relations

$$\delta'(x) = -\frac{\delta(x)}{x}$$

$$\delta(-x) = \delta(x) \tag{2.85}$$

we obtain

$$n(E) = \frac{\alpha\beta m A}{2\pi} \sum_{s=0}^{\infty} \sum_{n=0}^{\infty} \frac{\delta[(s+\frac{1}{2})\hbar\alpha + (n+\frac{1}{2})\hbar\beta - E]}{[(s+\frac{1}{2})\hbar\alpha + (n+\frac{1}{2})\hbar\beta - E]} \tag{2.86}$$

We now consider three limiting cases of this result.

1) When the nonlocal harmonic oscillator potential goes to zero, which corresponds to the case $\omega \to 0$, $\Omega' \to \frac{\Omega}{2}$, $\alpha \to \Omega$ and $\beta \to 0$, equation (2.86) reduces to

$$n(E) = \frac{\Omega m A}{2\pi\hbar} \sum_{s=0}^{\infty} \delta[E - (s+\frac{1}{2})\hbar\Omega] \tag{2.87}$$

This is exactly the well known density of states of an electron confined in two dimensions under the influence of a homogeneous transverse magnetic field (16).

2) When the magnetic field approaches zero, $\Omega \to 0$, $\Omega' \to \omega$ and $\alpha = \beta \to \omega$. From (2.86) we obtain

$$n(\mathcal{E}) = \frac{m A \omega^2}{2\pi} \sum_{s=0}^{\infty} \sum_{n=0}^{\infty} \frac{\delta[(s+n+1)\hbar\omega - \mathcal{E}]}{[(s+n+1)\hbar\omega - \mathcal{E}]} \qquad (2.88)$$

which is the density of the states of a two dimensional nonlocal harmonic oscillator.

3) When all potentials approach zero, namely, $\omega = \Omega = \alpha = \beta \rightarrow 0$; one can verify that n(E) in expression (2.86) reduces to the constant density of states of a two dimensional free particle,

$$n(\mathcal{E}) = \frac{m A}{2\pi \hbar^2} \qquad (2.89)$$

From the application of the exact propagator for the case of no electric field it is shown that, when the nonlocal oscillator frequency approaches zero, Eq. (2.86) reduces to Eq. (2.87) which is the density of states of a free electron confined in two dimensions under the influence of a transverse magnetic field. It is known that the n(E) in the expression of Eq. (2.87) gives an experimental density of states of a two dimensional electron gas such as that on the surface of a semiconductor, for example a silicon field effect transistor, which is necessary for the observation of the quantized Hall effect. The density of states expression in Eq. (2.86) which includes the nonlocal harmonic oscillator potential is seen to consist of the sum of numerous Dirac delta functions with difference amplitudes. The model presented in this manner is a rather idealized one and the n(E) expression does not show the behaviour of broadening beyond the Landau levels due to a disordered potential. To obtain a reasonable density of states including gaussian broadening we must go beyond the zeroth order cumulant approximation. The procedure for the calculation of the cumulant approximated propagator will be found in chapter III.